DEVELOPING ALTERNATIVE SCDDP IMPLEMENTATIONS FOR HYDRO-THERMAL SCHEDULING IN NEW ZEALAND

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by Rosemary Anne Read

University of Canterbury

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2. Abstract

In a hydro-dominated system, such as New Zealand, the continual improvement and development of effective optimization and simulation software to inform decision making is necessary for effective resource management. Stochastic Constructive Dual Dynamic Programming (SCDDP) is a technique which has been effectively applied to the New Zealand system for optimization and simulation. This variant of Dynamic Programming (DP) allows optimization to occur in the dual space reducing the computational complexity and allows solutions from a single run to be formed as price signal surfaces and trajectories. However, any application of this method suffers from issues with computational tractability for higher reservoir numbers. Furthermore, New Zealand specific applications currently provide limited information on the system as they all use the same two-reservoir approximation of the New Zealand system. This limitation is of increasing importance with the decentralization of the New Zealand electricity sector. In this thesis we develop this theory with respect to two key goals:

- *To advance the theory surrounding SCDDP to be generalizable to higher reservoir numbers through the application of the point-wise algorithm explored in R. A. Read, Dye, S. & Read, E.G. (2012) to the stochastic case.*
- *To develop at least two new and distinct two-reservoir SCDDP representations of the New Zealand system to provide a theoretical basis for greater flexibility in simulation and optimization of hydro-thermal scheduling in the New Zealand context.*

3. Abbreviations/Glossary

HVDC – High Voltage Direct Current

CDDP – Constructive Dual Dynamic Programming

SCDDP – Stochastic Constructive Dual Dynamic Programming

SDDP – Stochastic Dual Dynamic Programming

SRMC – Short Run Marginal Cost

DSS – Demand Surface for Storage

DSR – Demand Surface for Release

DSS' – Demand Surface for Storage and Release

MWV – Marginal Water Value

LDC – Load Duration Curve

4. Introduction

Reducing the cost of electricity and maximising the benefit to New Zealanders from our electricity generation resources is a perpetual and highly politicized concern. A significant aspect in to provide sustainable lower prices is the development of models and reservoir release and storage guidelines so that water storage potential can be used to aid the lowering of overall generation cost. In practice it is necessary that any such release strategies are tempered by the requirements of water for environmental, agricultural and in some cases recreational purposes. However, a useful source of information in developing usage and development policies for hydro-electric generation in the New Zealand context is the development of more effective hydro-thermal scheduling tools. This thesis is aimed at examining, redeveloping, and developing more effective and consolidated representations of the algorithm which underlies several of the tools which are reputedly still in use as one of the bases for reservoir release decision making.

The algorithm that is the basis of this thesis is now known as Stochastic Constructive Dual Dynamic Programming (SCDDP). We note that it has undergone a number of name variations in different publications of the algorithm, and in the literature review below we will attempt to clarify some of the publications that could be attributed to this technique.

This algorithm has been used as the base optimisation component of a number of industry and commercial implementations that have been developed for the New Zealand hydro-thermal scheduling problem. New Zealand based publicly funded incarnations include the RESOP module of SPECTRA and a market power based model entitled RAGE/DUBLIN. The technique also forms the basis for the Norwegian model ECON BID (E. G. Read, and Hindsberger, M., 2010).

There are two key focus areas for this thesis:

- *To advance the theory surrounding SCDDP to be generalisable to higher reservoir numbers through the application of the point-wise algorithm explored in R. A. Read, Dye, and Read (2012) to the stochastic case.*
- *To develop at least two new and distinct two-reservoir SCDDP representations of the New Zealand system to provide a theoretical basis for greater flexibility in simulation and optimization of hydro-thermal scheduling in the New Zealand context.*

The first of these focus areas is in response to one of the primary limitations of previous SCDDP implementations: they have been limited to two reservoir models.[1] Although this limitation is partially due to the prospect of issues with computational tractability the primary reason for this is that SCDDP has been defined in terms of expected 'shapes' and 'guidelines'. This spatial representation is not readily generalisable to higher reservoirs and it is primarily this limitation that we attempt to overcome in this thesis through the use of key 'points' or 'corners' from which surfaces and objects can be reproduced without the same level of visualisation.

The second of these focus areas attempts to extend the visualisation of the New Zealand reservoir space beyond that used for the initial SCDDP development. Previous implementations contain a two-reservoir configuration in which there is a single storage facility or reservoir modelled in each island and these are joined by a capacity constrained link. These implementations also only allowed for the inclusion of thermal generation in a single one of these islands. Consequently developing alternative representations in this thesis is intended to be exploratory development rather than a direct comparison of the benefit of each configuration. As part of this exploratory framework the 'double filled' Load Duration Curve (LDC) was developed to represent thermal generation occurring in both islands.

The conceptual frameworks developed throughout this thesis are designed as a consolidation and generalisation of the SCDDP algorithm to enable the further

[1]ECON BID at face value appears to be incorporate higher reservoir numbers, however the SCDDP aspect of this model still only operating over two 'reservoirs' where the second 'reservoir' is an aggregation of all other storage potential in the system. This will be discussed further in the Literature Review section.

development of this algorithm beyond the current two-reservoir limitations. Further to this the exploration of alternative reservoir configurations will provide a framework for further exploration of SCDDP reservoir configurations of the New Zealand system. These advances on the academic conceptualisation of SCDDP may in time facilitate a broader conceptualisation of methods of reservoir management in the New Zealand context. It is our hope that this research will enable more informed consideration of decision-making tools to help maximise the benefit to be derived from reservoir storage and minimise the costs associated with electricity generation in New Zealand.

5. Literature Review

The core focus of this thesis is the utilization of Stochastic Constructive Dual Dynamic Programming (SCDDP) to produce information about the value of water storage and release in a New Zealand based hydro-thermal scheduling problem. The academic context surrounding this thesis subsists of a number of overlapping categories and research focus areas. In particular this topic spans research into the reservoir management problem and the ongoing development of simulation and optimization of large-scale hydro dominated electricity systems, associated developments in stochastic dynamic programming based methodologies, and in the midst of this, a core component is the gradual evolution New Zealand based electricity system modeling techniques and applications. Each of these categories and the surrounding literature will be considered in the order as described. This reflects moving from more general areas of ongoing development that cannot be fully given justice given the brevity of this review to an intense focus on the environment that continues to dominate the evolution of SCDDP as a technique. A particular area of secondary focus in considering the New Zealand context will be the development of other hydro-thermal scheduling research tools, most notably SDDP, to gain broader insight into reservoir management in the New Zealand system.

5.1 The Reservoir Management Problem

Reservoir management techniques span the management, control and development planning for the storage and release of resources for various consumptive and non-consumptive purposes. These include single and multi-reservoir systems of various complexities. For hydro reservoirs consumptive purposes include the supply of irrigation and urban and industrial water supply. Non-consumptive purposes generally include electricity production and the reduction of water level variation in environments where there exists specific flood control, ecological or recreational requirements.

The set of decision support techniques applied to this context is large and spans the entire life-cycle of the reservoir, from initial planning to decommissioning. A fairly broad summary of simulation and optimization techniques is established Rani and Moreira (2010) with only cursory coverage of each key technique. Rani and Moreira (2010) differentiate broadly between optimization, simulation and simulation-optimization models. This distinction is somewhat arbitrary given the complexity of most reservoir management problems. The application of mid-term or long-term reservoir management optimization techniques to a simplification of a reservoir management problem is frequently the basis for simulation models. However, Rani and Moreira (2010) is the most comprehensive of recent perspectives on the development of these techniques.

Other earlier key state-of-the-art reviews include Yeh (1985), R. A. Wurbs (1993), and J. W. Labadie (2004). Yeh provides an intensive survey of the application of Linear Programming (LP), Dynamic Programming (DP), Non-Linear Programming (NLP) and simulation based techniques and the underlying mathematics. The majority of these techniques were at that time still in early developmental stage and correspondingly Yeh primarily establishes the mathematical and theoretical underpinnings of a generic application of these systems analysis techniques. This usefulness of this detailed approach is augmented by the utility focused approach adopted in Wurbs. To be clear, there are two key reviews in which R. A. Wurbs is noted as an author. The earlier of these is R.A. Wurbs et al. (1985), a review which largely parallels the review performed by Yeh. Whereas R. A. Wurbs (1993) builds on these earlier reviews through an exploration of specific models which have been developed and a sound basis for developing a model. The focus of Wurbs is on evaluating the advantages, limitations and assumptions of key decision support methods in order to provide a clear framework to assess the suitability of a technique to the particular reservoir management needs of an individual system. Particular considerations are the purpose of the reservoir, the availability and applicability of generalized models and the descriptive or prescriptive nature of the results. The core underlying theme in J. W. Labadie (2004) is the ongoing inability for operational models to capture the real system characteristics. A wide range of model types have been developed. Key variants include optimal control, LP, Network flow optimization, NLP, both discrete and differential DP and multi-objective optimization. Extensions of the above techniques to incorporate stochasticity are also considered.

In addition to these well-established methods Rani and Moreira (2010) also reviews advances in evolutionary computation, fuzzy logic, neural networks and combined simulation and optimization techniques applied to the reservoir management problem. Rani and Moreira (2010) also mentions as a separate set of techniques 'Large Scale Optimization Techniques' and identifies Stochastic Dual DP (SDDP) and Constructive DP as two examples alongside Neuro-DP and Reinforcement Learning (RL). All of these methods are intended to build on the benefits of Stochastic Dynamic Programming (SDP) applied to the reservoir management problem however each method is distinct in the approach taken to overcome the limitations inherent in SDP. Another technique undergoing development using the same base principles is Approximate Dynamic Programming (ADP) in Powell (2007) and variants thereof including that developed in Suvrajeet and Zhihong (2014). There are no publications to date which explicitly apply ADP or it's variant to the reservoir management problem as yet. However the similarities between this technique and Neuro-DP imply that such an application would be an intuitive area for further development.

5.2 Large Scale Optimization Techniques developed from Stochastic Dynamic Programming

The intuitive advantages of SDP for reservoir management are well-established throughout academic literature on the use of reservoirs. These advantages are perhaps best defined in J. W. Labadie (2004)’s discussion of the nature of the operational reservoir management problem. That is, it is a series of time-delineated decisions (dynamic) to release or store water based on a set of assumptions about future (stochastic) inflow probabilities. This formulation is inherently apt for a stochastic dynamic program as the problem formulation can be readily separated into time-delineated sub-problems. Given a certain assumption of stochasticity it would be most beneficial to develop solutions for the entire set of possible reservoir states such that as this stochasticity resolves for each sub-problem a decision can be made to maximize future benefit. However, the large-scale nature of the hydro-thermal scheduling problems severely curtails the ability of effective SDP's to be formed over all storage levels for multiple reservoirs. The complexity in the process is further added to where correlations between stochastic inflows are assumed between time periods. This has resulted in considerable research into successive approximation methods to find equilibrium optimal policies (Dreyfus & Law, 1977). Successful single reservoir SDP reservoir management implementations can be found in Huang, Harboe, and Bogardi (1991), Stedinger, Sule, and Loucks (1984), and Vasiliadis and Karamouz (1994). Along with a small number of very specific multi-reservoir applications such as Tejada-Guibert, Johnson, and Stedinger (1995). Typically sampling or estimation methods have been used to enable computationally tractable SDP models to be produced. Of particular interest is the approach by Labadie to develop optimal storage 'guidecurves' for a single reservoir SDP problem (J. Labadie, 1993a) based on the inverted SDP function. This 'guidecurve' approach bears similarities to the basis for SCDDP described below.

In order to develop solutions for large-scale reservoir management there are three key variants of SDP that have been particularly successful. The techniques of RL, Neuro-DP, ADP and variants thereof take a 'learning' approach. This is primarily based on the successive exploration of different release policies to facilitate a learning process. These techniques enable pockets of 'good' solutions to be explored and exploited. However, little information is available on the wider solution space. In contrast SDDP (established in Pereira (1989) and Pereira and Pinto (1991)) is based on SDP techniques by building up information to provide a solution to the dual of a dynamic programming formulation. This enables system complexity to be readily taken into account. SDDP formulations focus in developing a particularly good solution for the first period and then developing appropriate decision rules for later periods based on this solution. SDDP is perhaps the most commercially successful of these techniques and variants have been developed for the New Zealand system. In contrast SCDDP deals directly with the dual of the problem, 'constructing' optimal dual solutions directly to define operating 'guidelines' or policies over the complete state space and planning horizon. This technique has been developed primarily in the New Zealand context and a more comprehensive discussion of the formative academic and industrial literature surrounding it will be engaged in below.

5.3 New Zealand Hydro-thermal Scheduling models

Following on from early single-reservoir tools, SCDDP was one of the earliest two-reservoir models to be developed in the context of the New Zealand system. The precursor literature around the optimal operation of power systems was developed by E. G. Read (E. G. Read, 1979, 1984, 1985a, 1985b, 1985c). In these the economic basis for SCDDP and the potential of utilizing the dual space to create release guidelines and information for planning is established. These ideas were consolidated into the first incarnation and corresponding implementation entitled 'PRISM' of SCDDP in Winter (1987) designed for planning future generation development. This was developed into the RESOP module of 'SPECTRA' and used to manage reservoir releases which is discussed in J. Culy, Willis, and Civil (1990). Anecdotal evidence indicates that this early SCDDP incarnation is still in use in New Zealand reservoir management today. Developments since have been sporadic but developments include E. G. Read and Yang (1999) which extended the base SCDDP algorithm to incorporate inflow correlation, which extended the use of the algorithm to the coal stockpiling problem, E. G. Read and George (1990) extending the deterministic version of this technique (CDDP) for use in generalized stockpiling problems. An essentially identical technique to this deterministic variant of SCDDP was developed independently in Bannister and Kaye (1991) which developed by later literature into a multi-dimensional deterministic variant. T. J. Scott and Read (1996) adapted SCDDP to include components of gaming in a market situation and Kerr, Read, and Kaye (1988) considered the implications of risk aversion. These concepts were then further developed by Craddock et al. (1998) to include gaming and risk aversion as a model 'ECON' which is an aspect of ECON BID. The ECON BID model is described in E. G. Read and Hindsberger (2010) and uses reservoir aggregation in order to apply the two reservoir module to a multi-reservoir system. Read and Hindsberger consolidates much of the literature on SCDDP and clarifies the interplay between different streams of development of the algorithm. The more recent publications of R. A. Read et al. (2012) utilize a single reservoir SCDDP model to develop solutions for use in water markets. Read, R. A. outlines the underpinning development for CDDP of a generic ‘pointwise’ algorithm which will be extended to an SCDDP ‘cornerwise’ algorithm in the context of this thesis with New Zealand based applications to a variety of possible New Zealand reservoir configurations (R. A. Read, Dye, S. & Read, E.G., 2012).

6. New Zealand Electricity Sector

The New Zealand Electricity sector has a number of unique characteristics that have rendered hydro-thermal scheduling to be a particularly important aspect in the ongoing assessment of development, market behaviours and security of supply issues.

From a geographic and development perspective there are significant challenges that are faced by the New Zealand system. The dominance of hydro-electric generation in the system is coupled with a relatively low storage capability proportionate to the annual inflows. This makes the management of the existing reservoir capacity of paramount importance in maintaining a secure supply of electricity throughout the year. Furthermore the significant deviations in inflow levels between years can create issues due to corresponding large differences in level of load that can feasibly be met by hydro-releases in different years. Where inflow levels are higher than the predicted levels then there is a risk of storing unnecessarily high quantities of water. Water may be retained when it could otherwise be released in a given time period to lower thermal generation costs in system. This would increase electricity costs over the year significantly in New Zealand, raising the price particularly in the higher inflow periods. Conversely where inflow levels are lower than the predicted levels then there is a risk of shortage, this would drive high electricity prices in the low inflow season as less of the seasonal load is being filled by inexpensive hydro-power. Both of these would result in significant seasonal price fluctuations. However, more significantly, under perfect competition or a central dispatch model, both of these scenarios would also result in a higher average electricity costs, and higher electricity prices in general for the New Zealand system.

In practice these inefficiencies would occur as, in hindsight, water was not being used where it was most valuable. The value of stored electricity is equal to the marginal price of the thermal or other generation that the hydro-electric generation is used in the place of. There are fewer renewable resources to meet the load during a low-inflow season, consequently larger quantities of the thermal capacity is active, and so the more expensive thermal facilities are run to fill the load. Hence were there no storage the marginal thermal in a low-inflow season would always be more expensive than that in a high inflow season. If water can be stored to supplement this generation, the value of replacing a more expensive thermal is higher than the value of replacing a less expensive thermal. This increases the over-all efficiency of the system. In New Zealand, the careful management of our hydro-electric storage facilities can cause significant improvements in the efficiency of our electricity system and correspondingly lower the price. These concepts are explored in greater detail in the section on single reservoir CDDP below (see page 35).

Likewise, however, the unique features of the New Zealand system also create significant difficulties in developing any appropriately detailed hydro-thermal scheduling model. In the first instance these issues primarily arise from the configuration of the New Zealand system. There are inherent difficulties in modelling the New Zealand hydro-thermal system as there are two main islands in New Zealand. Both these islands are major sources of both load and generation, and they are connected by a capacity constrained, loss-adjusted HVDC link. This allows some inter-island transfer. The link was deemed particularly necessary due to the majority of the hydro-electric storage and generation capacity being centred in the South Island, whereas the high load is primarily centred in the North Island. Conversely though, in dry years the geographic imbalance between generation and load is effectively reversed and the North Island thermal capacity is used to supply a component of the South Island load.

The New Zealand HVDC link is used to connect the Bunnythorpe (BPE) node to the Haywards (HAY) on the transmission network. There have been 3 commissioned links over the course of the HVDC link's existence. The initial link was a bipolar 600 MW link. This was later was paralleled to form a single pole (Pole 1). At this stage a newer pole (Pole 2) was commissioned increasing the link capacity to 1040MW. In May of

2013 a new pole (Pole 3) was fully commissioned which increased the capacity of the link to a 1200 MW configuration (Trans Power New Zealand, 2008, 2013).

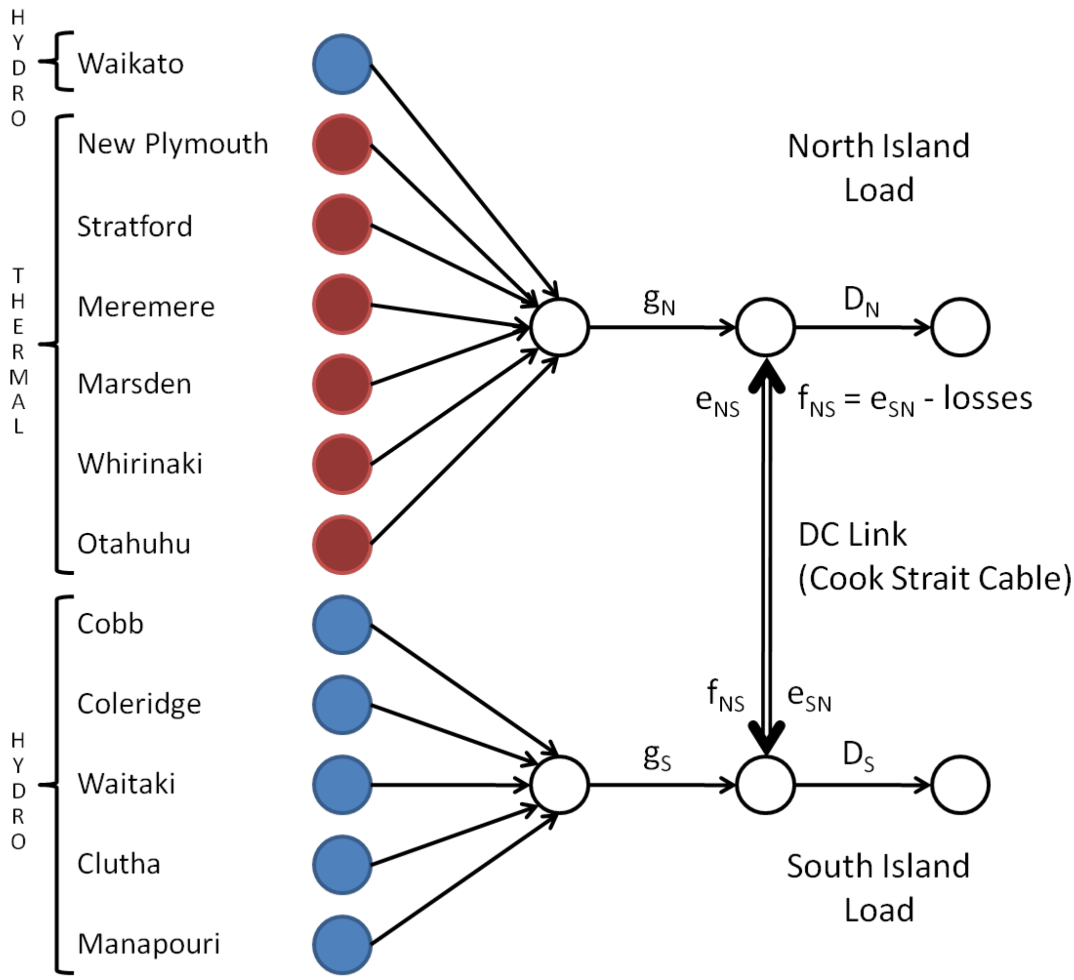


Figure 1: Simplified diagram of NZ system with HVDC link

The HVDC link means that any moderately realistic model of the New Zealand system requires at least two distinct regions with a link between them. In the case of hydro---thermal scheduling this has generally resulted in the development of two---reservoir models with a constrained link between them. This has included such models as PRISM, SPECTRA, and RAGE based on the CDDP concepts as described in academic terms in the 5. Literature Review above, and with a more detailed explication of the algorithm in the section on single reservoir CDDP (see page 35) and the sections extending this to two reservoirs using SCDDP (see page 119) below. There were initially a number of single reservoir models of the New Zealand system used for planning purposes in the Department of Energy, however these were largely determined to be inadequately detailed representation of the New Zealand System for both long and short---term reservoir scheduling due to the high probability of different storage policies being necessary for each island (Lermit, Barr, Garner, Rhoades, & Read, 1989). Two reservoir models are considered as the standard minimum level of detailed required for modelling the New Zealand system. These two---reservoir models have been generally CDDP based. However, later explorations into possible techniques to apply to the New Zealand hydro---thermal scheduling problem considered CDDP based models to have limited further development potential (Halliburton, 1994). The limitations identified were that they aggregate the reservoirs and inflows in each island, and cannot fully take into account transmission limits, spinning reserve requirements, ramp limits, temporal inflow correlations, and arguably tributary inflows (Halliburton, 1994). In response to these limitations variants of SDDP which take into account higher reservoir limits have also been adapted to the New Zealand system. The advantages and disadvantages of each of these techniques is discussed in detail in the 5. Literature Review above.

However, the focus of this paper is developing the SCDDP algorithm described below for one and two reservoir models of New Zealand. In this section we will primarily focus on the New Zealand system features, and possible adequate one or two reservoir representations of this. The traditional twoR   reservoir approach taken for the New Zealand system is of an aggregate reservoir for each island as represented in Figure 2 below.

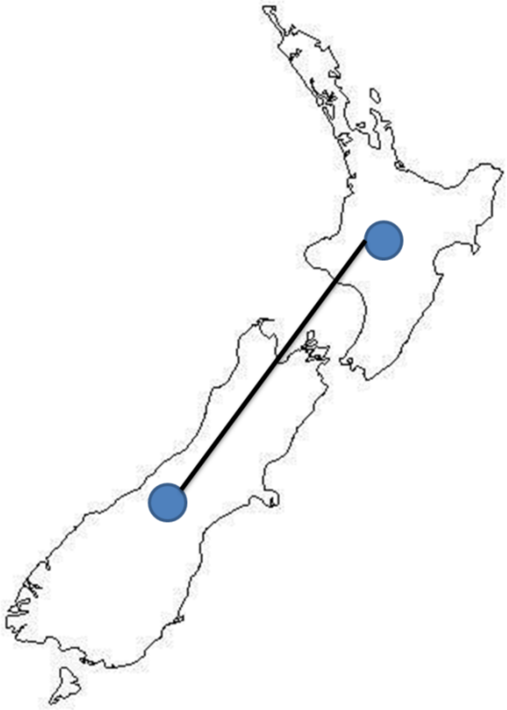


Figure 2: Modelling the North Island and South Island as reservoirs

In this representation there exists a single North Island reservoir and a single South Island reservoir. Each island has both distinct load requirements and alternative generation sources. These two islands are linked by a capacity constrained HVDC link. Consequently any electricity generated in either island can be transferred to fulfil demand in the other island up to the HVDC line capacity. Electricity transferred along the HVDC line is subject to losses. This representation allows for the development of release guidelines for nearby reservoirs using heuristic methods. In effect hydro-generation is in two clusters, and this representation gives a point of information about each cluster allowing useful release policies to be developed for each island. However as the cluster of hydro-generators in the South Island is larger, with three large hydro schemes in the South Island and the annual output from the Waitaki scheme alone exceeding all major annual hydro-generation output from the North Island. It is likely that the application of SCDDP to the New Zealand system could be moderately improved by the exploration of alternative configurations.

Models based on the configuration shown in Figure 2 have been used with some efficacy. Nonetheless, it is debatable whether this base configuration constructs the best representation of the New Zealand system. Ideally the solutions produced from various configurations and techniques applied to the New Zealand system would be objectively evaluated. However, given that there is no objective, truly optimal solution that has been produced for the entire New Zealand system to compare against, then at best models can only be compared in terms of their differences and usefulness given historical system behaviour. Further to this, no comprehensive benchmarking test has been done over a set of likely New Zealand inflows to resolve the question of solution quality, and this is beyond the scope of the current review. To be clear, this thesis develops models for different configurations that may be useful, however the scope of this thesis does not include benchmarking analysis comparing the solution quality of these different configuration.

Primarily the standard configuration shown in Figure 2 is limited in that it fails to take into account that comparatively the levels of generation potential stored in North Island hydro-reservoirs are low, compared to those in the South Island. This imbalance means that release decisions made for the entire North Island are likely to have less impact on the New Zealand system on average than those made for individual key storage lakes and schemes in the South Island. In terms of generation volume, 72% of average annual hydro-electric generation is based in the South Island ("Generating Station List as at September 2012," 2012).

In response to these limitations, it may be unnecessary to have both distinct regions represented as reservoirs when modelling the New Zealand system. An alternative configuration, which may be appropriate for the New Zealand system, is based on modelling the two hydro-electric schemes with the largest total mean generation potential. This could either be applied to New Zealand data through applying CDDP to the largest reservoir in each scheme or by aggregating reservoirs. The two hydro-schemes with the largest mean generation potential are the Clutha (359 GWh) and the Waitaki Scheme (1744 GWh) - both southern hydro schemes (OPUS, 2010). The generation potential stored in the reservoirs of each schemes conjoined considerably exceeds the North Island generation potential storage (422 GWh) (OPUS, 2010). Such a representation of the New Zealand system can be seen in Figure 3 below.

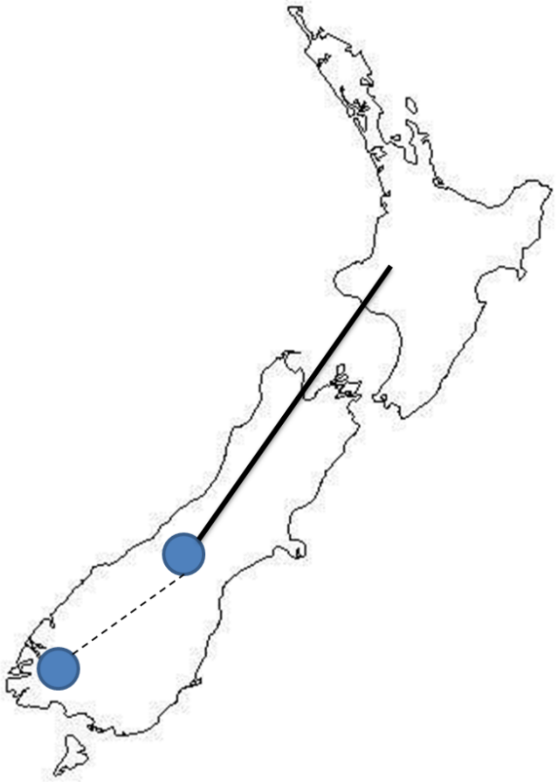


Figure 3: Modelling the Waitaki and Manapouri schemes as individual reservoirs

This configuration would be advantageous as it would allow there to be information points more closely proximate to the two key clusters which have a sizable influence in the New Zealand system. Release strategies for both the North Island, and the remainder of the South Island could be developed heuristically either independently or based on the information produced from the SCDDP optimization. However, it is also possible that any release strategies in the North Island heuristically developed from the South Island generation and inflow information points would be less useful due to the distance between the generators there and the points of information the heuristics are based on.

Another possible useful representation would be to model the Waitaki scheme alongside the Manapouri generation facility, however the SCDDP component would be mathematically identical to the Clutha and Waitaki configuration. The only difference would be in the reservoir specific information.

A significantly different representation however, would be based on modelling the two largest reservoirs in New Zealand. The mean potential generation values for the two largest South Island storage lakes, Lake Pukaki (1130 GWh), and Lake Tekapo (614 GWh) are both larger than the combined mean potential generation value of both major North Island reservoirs combined (422 GWh) (OPUS, 2010). Likewise the Manapouri reservoirs with Lake Manapouri’s mean potential storage of 113 GWh and Lake Te Anau’s mean potential storage at 213 GWh are dwarfed by even one of these Waitaki-based storage lakes (OPUS, 2010). This suggests that modelling these Lake Pukaki and Lake Tekapo reservoirs could be a fairly accurate way to capture the best-use potential of the majority of hydro-electric storage potential for New Zealand. However this particular reservoir configuration presents a challenge in developing models as both these reservoirs are on the same river chain. This means that the upstream releases will impact on the storage levels of the downstream reservoir. In the simplest form this reservoir configuration could be modelled as depicted in Figure 4 below.

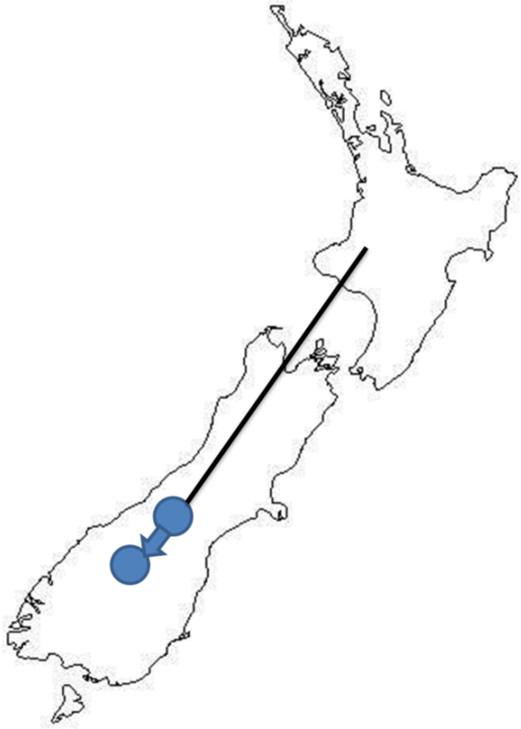


Figure 4: Modelling Pukaki and Tekapo reservoirs

In this representation water released from one reservoir arrives in the downstream reservoir – these are effectively reservoirs in series. Generation sources in both islands would still be able to transfer through the HVDC link to supplement local supplies when appropriate price signals occurred.

This representation would provide explicit information on the expected value of water in the two reservoirs that most significantly impact the New Zealand system, allowing better management of a high proportion of the system. Again release strategies for other reservoirs in both islands could be developed heuristically either independently or based on the SCDDP information. However, a two reservoir southern lake representation is less useful in the context of developing points of information for heuristic extrapolation in both the North Island and on other river systems. This is because the generation and inflows are likely to be based on very localized underlying factors.

Notably, this depiction does not take into account the full complexity of the Waitaki system. There are a number of generation facilities that comprise the Waitaki system. As can be seen on the diagram below there is the possibility for releases from Lake Tekapo to be used for generation and then flow through to Lake Pukaki. However, there is also the option of allowing some of the outflow from Tekapo to be redirected to be inflow directly to the Benmore hydro-electric generation facility. Likewise Lake Pukaki flows can be directly released to Benmore or can be used for generation at Ohau A, B and C.

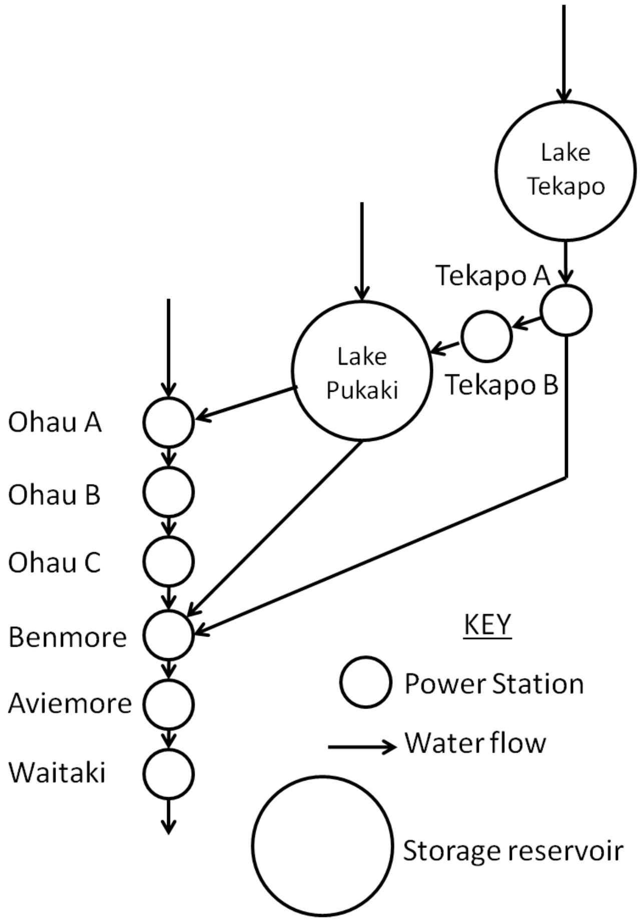


Figure 5: A simplified model of the Waitaki system

Limited academic attention has been given to the issues presented in representing the Waitaki system accurately. However, research in this is primarily been focused on the intra-week problem that is beyond the scope of this thesis. We note that ongoing work at the University of Auckland has been considering the extension of an SDDP model of the New Zealand system to take into account the existence of reservoirs in series, if not the added complexity of the full Waitaki system.

Alongside the geographic issues, the market–based structure of the New Zealand system results in a requirement for good modelling capability. Modelling can be used as a basis for market participants to make informed bids, and correspondingly for effectual market monitoring. The New Zealand system has moved in the last 30 years from a centralised system to a wholesale electricity market ("Energy Data File," 2012). There are five major generators with most generators operating as vertically integrated 'gentailers' with retail divisions. The national grid is a natural monopoly owned by Transpower, as system operator they hold responsibility for ensuring the security and quality of supply. However, the decentralisation has been shown not to have significant impact on the core principles of applying SCDDP to the New Zealand system (T. J. a. R. Scott, E.G., 1996). Under the centralized system these hydro-thermal scheduling tools were primarily used for long-term and mid-term scenarios to test the impact of different centrally planned developments to the New Zealand system, or to develop centralised dispatch guidelines. With the movement of the New Zealand electricity generation sector to a market structure both generation companies and the Electricity Authority of New Zealand have used hydro-thermal scheduling models. Generators use hydro-thermal scheduling to isolate and optimize releases from assets over which they have control, or to determine commercial strategies based on predictions of competitors actions. The Electricity Authority uses hydro-thermal scheduling as an aspect of its market monitoring function. In this role, the Authority may investigate a market participant bidding a generation facility's capacity at a particularly high price. Bidding at a consistently high price can be of concern as it may indicate an attempt to exercise market power in the wholesale electricity market. However, for a hydro-electric generator a high price may also indicate risk-aversion to shortage and reflect a belief that the inflow levels will be low in a given year. Hydro-thermal scheduling tools can be used to determine a reasonable range for the facility's marginal water value, and corresponding an appropriate range of prices for generation. Although this application of hydro-thermal scheduling cannot confirm the use of market power, it can be an indicative measure. A further possible application of such tools is in the valuation of generation assets, or potential generation projects. However, it is notable that all of these market functions are likely to be better informed through the consideration of a number of different reservoir configuration assumptions. Furthermore, where possible, increasing the number of reservoir configurations that are modelled improves the quality of the information that can be developed by market participants for the operation of the subset of reservoirs under their control.

The New Zealand hydro---thermal scheduling problem is both deeply important to the ongoing stability of the New Zealand electricity supply and any market developments. Despite the difficulties presented by the geographic aspects of the New Zealand system, a number of techniques have been applied to develop useful guidelines as a basis for reservoir releases. Each of these has advantages and disadvantages which are outlined in more detail in the 5.
Literature Review above. Constructive Dual Dynamic Programming (CDDP) and variants thereof are amongst these techniques and the extension of this algorithm, and application to a number of New Zealand system representations will be the exclusive focus of the remainder of this thesis.

7. Constructive Dual Dynamic Programming (CDDP)

Constructive Dual Dynamic Programming (CDDP) is the key technique from which Stochastic Constructive Dual Dynamic Programming (SCDDP) emerges. This deterministic variation of Dynamic Programming (DP) was developed as one of many responses to the limitations on traditional DP.

DP is a technique that fundamentally relies on breaking the larger problem down into a series of sub-problems applied to a discrete representation of the state (primal) space. Where the state space is naturally discrete and has a small quantity of discrete states then DP can be a very effective method. However with every linear increase to the dimension of the state space, the complexity of the DP model increases. Consequently, where a problem has a large number of values needed to represent the discrete states, or where the state space is continuous and represented by a comprehensive, arbitrary set of discrete points, the problem can rapidly become computationally infeasible. This is often described as the 'curse of dimensionality' (Yakowitz, 1982), a phenomena which plagues Linear Programming (LP) and DP alike.

To find the optimal DP solution from a given starting state, DP requires solutions for each possible system state. In a multi-dimensional problem, such as a multi-reservoir hydro-thermal scheduling problem, this implies determining the optimal decision at each stage for each feasible combination of reservoir states which can arise in the multi-reservoir problem given a defined set of end states. Thus the addition of each reservoir increases the complexity of this calculation. Where the state space in one dimension is represented by a large number of values then increasing the dimensionality rapidly renders the problem computationally infeasible. CDDP has been developed in order to mitigate these limitations on DP through the application of discrete DP principles to problems that are discretised in the dual (marginal value) space. This method reduces the computational burden for those problems that are continuous or have large numbers of discrete points in the primal space and yet are defined by only a limited number of critical values in the dual space. The often arbitrarily discretised continuous state space of the primal can be replaced by the state space of the dual which is defined solely in terms of a discrete set of critical prices or values at which the solution changes(E. G. Read & Hindsberger, 2010). This formulation of the problem is advantageous as it allows a more compact representation. The more compact representation correspondingly reduces the computational burden. However, this simply reduces the computational effort of the problem with more granular data within the current dimensional space, rather than the complexity (the rate at which that computational effort will increase as dimensionality increases). Hence, in the process described above, each additional dimension will increase the state-space representation complexity exponentially. The compact structure ultimately can only forestall the inevitable computational tractability issues and allow for a small number of additional dimensions. As with all variations of DP, the 'curse of dimensionality' ultimately applies to CDDP.

The CDDP technique is primarily based on DP solved by backwards induction. DP based techniques are particularly useful where the problem structure can intuitively be broken down into multiple sub-problems, and the set of final system possible system states is known. An example of this is where a number of decisions must be made in a sequential order. Where the final state is known, then the known impact of each possible final decision in the system can be applied to the final state of the system. Deterministic DP assumes that this impact has a discrete set of possible values from which it originated. Consequently, through backwards induction the set of possible penultimate system states can be enumerated. The penultimate impact will then be applied to the state of the system at each of these enumerated outcomes. This process of moving backward through the system will continue until all the sub-problems have been addressed in a sequential order. Note that this sequential order is the opposite order to that which each choice will be made in practice. Once all sub-problems are addressed the full array of possible initial system states is identified, and the set of decisions necessary to produce the optimal final solution is determined by working forward through the enumerated outcomes until the final state is reached. Thus an optimal path is ascertained.

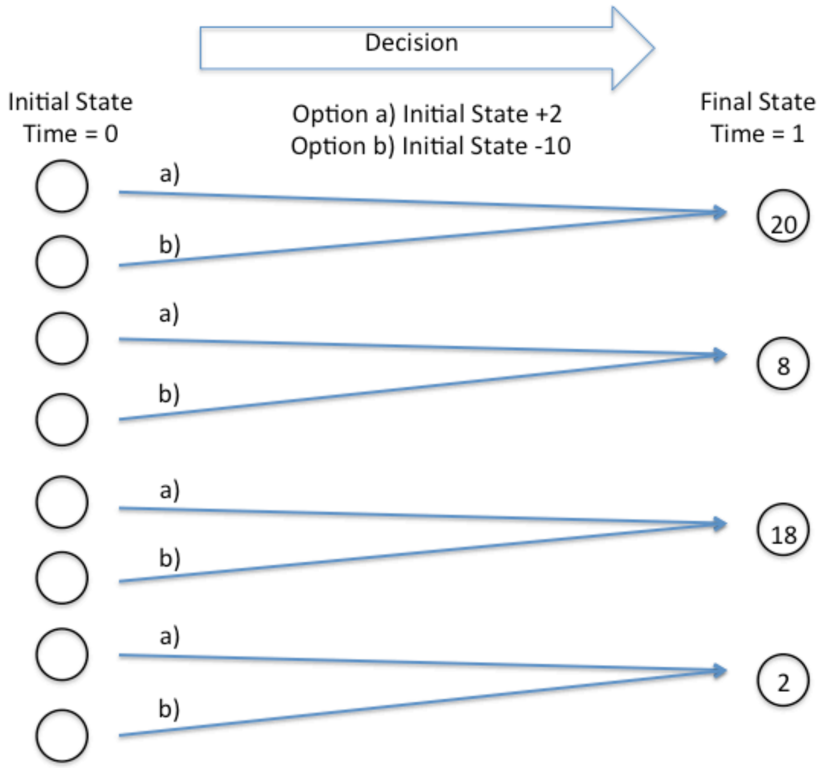


Figure 6: Final states and decision effects known

The above Figure 6 represents the situation in which the system operates over a single time interval and there is one decision to be made. In the above representation the possible final states are known, as are the consequences of deciding for 'Option a)' or for 'Option b)' given a particular initial state. As we know how the final state came into being we can rearrange the implicit algorithms as follows to discover the initial states that can result in these final states:

$$
\begin{gathered}
For\ Option\ a) \\
State_{Final} = State_{Initial} + 2 \\
State_{Initial} = State_{Final} - 2 \\
For\ Option\ b) \\
State_{Final} = State_{Initial} - 10 \\
State_{Initial} = State_{Final} + 10
\end{gathered}
$$

This then allows us to populate the possible initial states as is shown in Figure 7 below. Once this diagram is fully populated, then given the known initial state of the system a decision can be made in order to produce the desired final state. Notably this example does not delineate all the feasible states that are possible given the set of initial states. Instead we assume that the full discrete set of desirable final states is captured and consequently only the initial states that relate to these final states are worth considering.

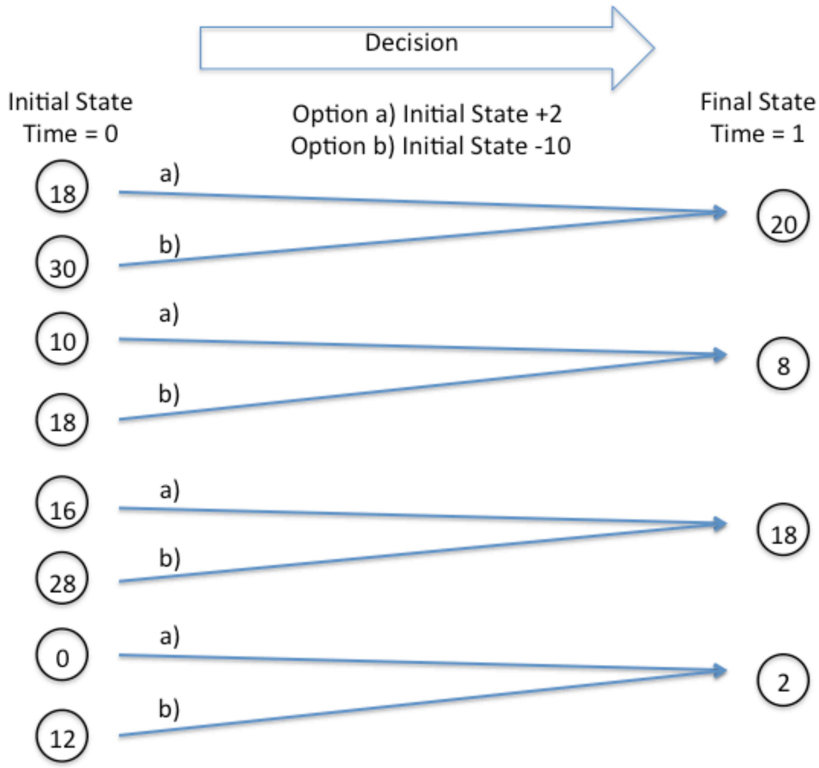


Figure 7: Set of initial states developed for each final state for each possible decision

CDDP solves problems through identical operations to DP solved by backwards induction. However, it is used where there exists a small set of points at which the dual/ price (or in this instance Marginal Water Value) state space changes. These different dual values are delineated as system states and the optimal path is ascertained through the application of the DP technique described above in the dual space. From this solution, the implications for the primal space will then be derived. CDDP is therefore an advantageous technique for problems where there exists only a small number of known constant marginal costs, such as in the hydrothermal scheduling problem described below (E. G. Read, and Hindsberger, M., 2010).

7.1 Key Surfaces

The technique of CDDP and stochastic variants thereof relies on the use of basic arithmetic to form relationships between functions that we will define in terms of three key surfaces. We have developed this new representation over a number of recent CDDP based publications including (Dye & Read, 2012) so as to better reflect the parallels between the hydro-thermal scheduling problem and the underlying economic principles of supply and demand. The simplicity of the arithmetic dealt with in the CDDP algorithm itself is key to the rapid computation times that have been noted for CDDP for one and two reservoir.

7.1.1 Demand Surface for Release (DSR)

CDDP is based around staged decision making, in this context the Demand Surface for Release (DSR) is a surface that represents for a particular decision stage the additional benefit that will be accrued by each incremental increase in the decision quantity. Where the decision relates to a single product this is the equivalent of the marginal value of an incremental increase in the quantity of product that is produced during that decision stage. A generic DSR for a single product is shown below.

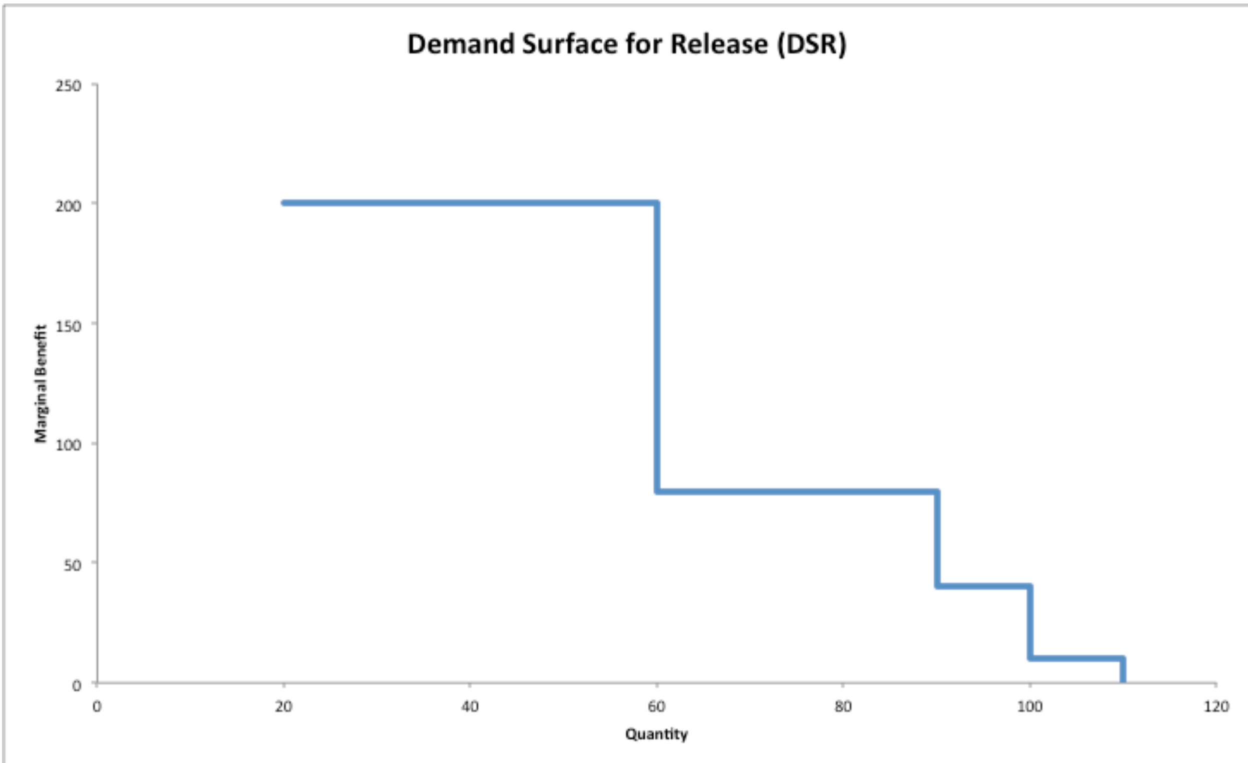


Figure 8: Demand Surface for Release (DSR)

The total value or benefit accrued by a particular decision quantity is the sum of the marginal values of all quantities up to and including the decision quantity. So where the marginal benefit function is piecewise constant the benefit function will be piecewise linear.

Where there a multiple substitutable products the marginal benefit of a particular product may be reduced by an increase in the production of another product. In this case the DSR represents for every product, the marginal benefit of each incremental product increase, given every possible decision level combination of every other product.

7.1.2 Demand Surface for Storage (DSS)

The Demand Surface for Storage (DSS) is a surface that represents for a particular decision stage the additional benefit that will be accrued by each incremental increase in the quantity stored. The quantity stored is the quantity that is not used in the current decision stage and is, instead, stored for use at a later decision stage. Generically speaking the DSS will take the same form as the DSR and where the resource is not replenished over the total decision horizon it is formed by the addition of all DSRs for future periods. This is equivalent to the addition of the DSR for the following period to the DSS for the following period:

$$DSS_t = DSR_{t+1} + DSR_{t+2} + \cdots + DSR_n$$
$$DSS_t = DSR_{t+1} + DSS_{t+1}$$
$$where\ t = Decision\ Stage\ and\ n = end\ of\ decision\ horizon$$

Algorithm 1

Note the addition here adds quantities at which the same marginal benefit is returned. Where the resource is replenished during the decision horizon then it is assumed that the quantity by which the resource is replenished will be used to fulfil the future DSR needs with the highest value. In this circumstance the DSS is formed by the addition the DSR of the following period and the subtraction of future inflows.

$$DSS_t = DSS_{t+1} + DSR_{t+1} - i_{t+1}$$
$$where\ i = inflows\ and\ t = Decision\ Stage$$

Algorithm 2

A further possible limitation on DSS formation is that there may be storage constraints that restrict the holding capacity of the decision-maker for future use.

This implies that at each decision stage any storage restrictions must be applied to the DSS for that future period.

$$DSS_t = \min\,(DSS_{t+1} + DSR_{t+1} - i_{t+1}, MaxStor)$$
$$DSS_t = \max\,(DSS_{t+1} + DSR_{t+1} - i_{t+1}, MinStor)$$
$$where\ i = inflows, t = Decision\ Stage, MaxStor = Maximum\ Storage\ and$$
$$MinStor = Minimum\ Storage$$

Algorithm 3

7.1.3 Demand Surface for Storage and Release (DSS')

The Demand Surface for Storage and Release is an intermediary surface that is used where inflows and/or storage limitation exist. This is the sum of the DSS and the DSR.

$$DSS'_{t+1} = DSS_{t+1} + DSR_{t+1}$$

Algorithm 4

Using Algorithm 4 as the intermediary stage, Algorithm 2 and Algorithm 3 above could be written as:

Algorithm 2 becomes:

$$DSS_t = DSS'_{t+1} - i_{t+1}$$

Algorithm 3 becomes:

$$DSS_t = \min\,(DSS'_{t+1} - i_{t+1}, Reslim_{Upper})$$
$$DSS_t = \max\,(DSS'_{t+1} - i_{t+1}, Reslim_{Lower})$$
$$where\ i = inflows, t = Decision\ Stage, MaxStor = Maximum\ Storage\ and$$
$$MinStor = Minimum\ Storage$$

Algorithm 5

These surfaces underpin the CDDP formulation and will be used with specific reference to the reservoir management problem below.

8. Constructive Dual Dynamic Programming Applied to the Reservoir Management Problem

In order to understand how CDDP is applied to the reservoir management problem we must first discuss the problem that CDDP is being used to solve. The fundamental processes are shown in Figure 9 below.

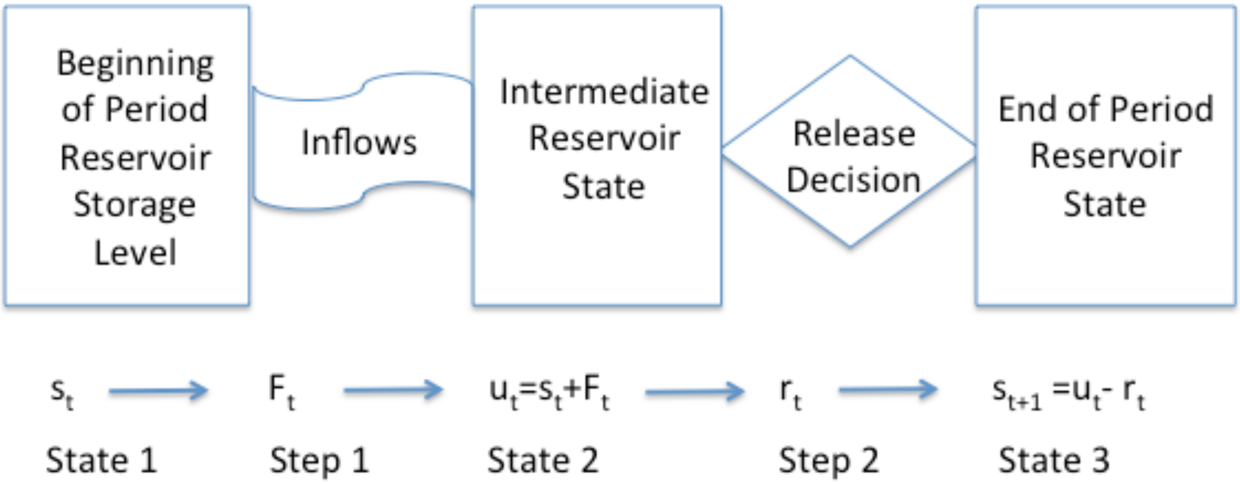


Figure 9: Reservoir Management Problem for a single time period

For a reservoir management the CDDP transition algorithm is:

$$s_{t+1} = s_t + F_t - r_t$$

Algorithm 6

where s_t is the storage level (state variable) at the beginning of period t, r_t the release and F_t the inflow. Our CDDP implementation divides the transition algorithm into two steps using the intermediate variable u_t representing all water available in the time period for storage or release:

$$u_t = s_t + F_t$$
$$s_{t+1} = u_t + F_t$$

Algorithm 7

The intermediate variable u is only for convenience, in practice we assume releases and inflows occur simultaneously and at a constant rate over the period. Therefore storage bounds are not applied to u.

The optimal release decision r_t is chosen to maximize total benefit. CDDP assumes the benefit from storage at the beginning of period t+1 is independent of the benefit from release in period t (except as linked by the transition algorithm). Rather than dealing with the benefit functions directly, single reservoir CDDP uses the marginal increase in benefit curves these are labelled demand curves due to their clear parallel with economic demand curves. These curves can be thought of as functions of marginal benefit increase (marginal value) as a function of water volume or the dual: water volume as a function of marginal value. These benefit curves extend to benefit surfaces for CDDP applications of two reservoirs and are described above as the DSR, DSS and DSS' above.

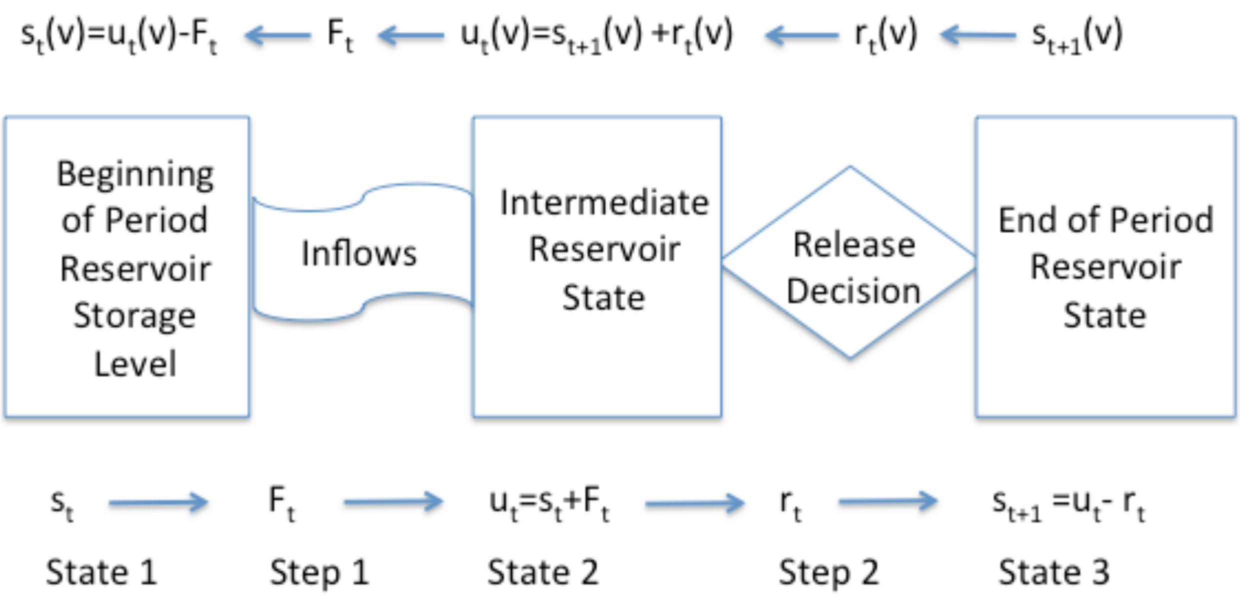


Figure 10: Single period Reservoir Management Problem

In order to solve the CDDP problem using backwards recursion we assume we have a known a DSS for start of period t+1 represented as storage 'demand' as a function of marginal value, $s^{t+1}(v)$, and a known DSR represented as release 'demand' as a function of marginal value, $r^t(v)$. The intermediate 'water demanded' given a marginal value v is given by:

$$u_t(v) = s_{t+1}(v) + r_t(v)$$

Algorithm 8

That is, simply add the appropriate representations of DSS and DSR over all marginal values. The intermediate 'water demanded' as a function of marginal value is referred to in this thesis as DSS'. To form the start of period t DSS, inflow is accounted for as:

$$s_t(v) = u_t(v) - F_t$$

Algorithm 9

But here the storage bounds must be applied so $s_t(v)$ is adjusted to $s_t(v)=Reslim_{Lower}$ for all marginal values v with $u_t(v) – F_t< Reslim_{Lower}$ and to $s_t(v)=Reslim_{Upper}$ for all marginal values v with $u_t(v) – F_t> Reslim_{Upper}$.

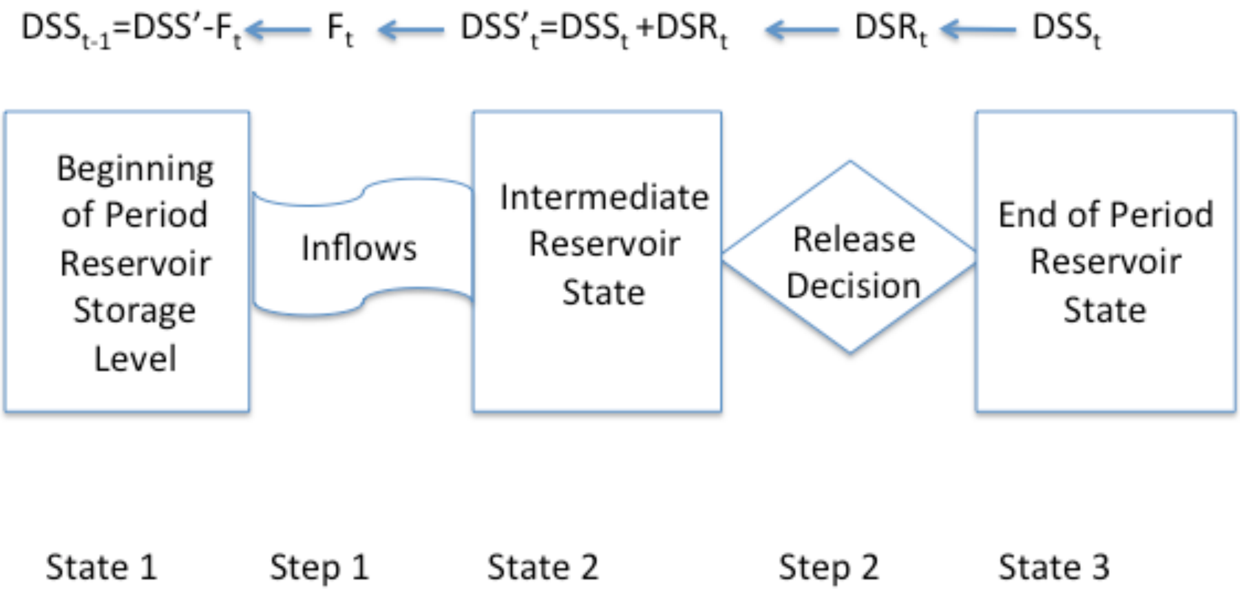


Figure 11:Single Period Reservoir Management Problem in backwards induction terms

Note that we make some assumption as to the likely marginal value of storage beyond the modelled horizon, that is we assume a function for $s_{t+1}(v)$ where t=final modelled period. An equilibrium function can be discovered through running the CDDP algorithm in a cyclic fashion setting $s_{t+1}(v)$ equal to $s_1(v)$ where t=final modelled period. The use of this method or a variant thereof would provide a more robust estimate for planning purposes. However, for this thesis we have simply assumed that $s_{t+1}(v)=r_t(v)$ in the instances tested as the primary goal is to successfully implement the CDDP algorithm for different configurations.

- The Marginal Water Value (MWV) values in the DSS are the values of v in $s_{t+1}(v)$. Each MWV is the marginal value associated with storing an additional increment of water for use in a later time period.

- The MWVs in the DSR are the values of v in $r_t(v)$. Each MWV is the marginal value associated with releasing an additional increment of water for use in the current time period.
- The MWVs in the DSS' are the values of v in $s_t(v)$. Each MWV is the marginal value associated with a unit of water available at the start of this time period. This unit of water may either be slated for release or storage. This depends on whether the unit of water initially comprised part of the DSS or the DSR for this time period.

In our consideration of the primal problem described above these surfaces are described as functions of the marginal value v. That is:

- DSS $\Leftrightarrow$ $s_{t+1}(v)$
- DSR $\Leftrightarrow r_t(v)$
- DSS' $\Leftrightarrow s_t(v)$

These represent water volume as a function of marginal value. However these same surfaces can be equivalently represented as marginal value as a function of water volume. For simplicity, we assume that reservoir storage and reservoir release is measured in equivalent energy terms:

- DSS $\Leftrightarrow$ $v_{t+1}(s)$
- DSR $\Leftrightarrow$ $v_t(r)$
- DSS'$\Leftrightarrow$ $v_t(s)$

For a single reservoir case the surfaces can be represented as MWVs associated with particular reservoir levels. The DSR for a time period represents the marginal value associated with using each additional unit of water for generation, this aligns with the description in the 7.1 Key Surfaces section. This marginal value is equal to the Short Run Marginal Cost (SRMC) of the generator would otherwise need to supply that unit of electricity. Where the next unit of water is used to reduce or prevent shortage then this value is equal to the theoretical value of lost load. The DSR for a time period could be stored a number of equivalent ways: as an arbitrary set of discrete MWVs with associated release quantities, as discrete release quantities, with their associated marginal values, or as in the single reservoir model of (Dye &

Read, 2012). In higher dimensions, though, it is more efficient to store the DSR as a discrete set of critical marginal water values, with their associated cumulative release quantities. These cumulative DSR quantities represent the maximum quantity that it is worthwhile to release where the marginal value of water is equal to a given generation facilities SRMC.

In the primal space the inclusion of storage means that if the unit of water were stored then it would be used to fill 'demand' in some later period. From the perspective of the dual function if a unit of water is stored then instead of accruing the MWV associated with an incremental release in this period that unit of water will accrue the MWV associated with an incremental release in a later period. So if the marginal value of releasing that unit in a future time period is higher than the marginal value of releasing that unit in this time period then the MWV of that unit of water will be equal to marginal value of release in a future time period. This unit of water would correspondingly be stored to meet that future 'demand'. For example, where the value of releasing the water in the later period is $1000, whereas the value of releasing the water in the current period is only $10 then by storing that water for the future period the solution value can be improved by $990. It is intuitive therefore that this unit of water should not be released now, unless the value of releasing in the current period is $1000 or more. Conversely, if the value of using the water in a future period was $1000, and the value of using water in the current period is $1100, then the water would be used in the current period. This trade-off between the future benefit and present benefit of a unit of water is applied to every unit of water that can be stored, for each period of time in the modelled timeframe. It is notable that where no storage or release bounds exist then a single MWV will exist for the entire modelled period. This is because any difference in MWV implies that storing more water from the period with the lower MWV to the period with the higher MWV will improve the overall benefit or vice versa. This means that where there continues to exist a difference in MWV then the solution cannot be optimal. It is inherent to this valuation method that units will cease to be stored for two possible reasons – either the reservoir is empty, or the value of releasing a unit in the given period is equal to the value of releasing a unit in any future period. As we have described the reservoir as boundless, this implies that there is no lower limit: the reservoir cannot be empty. Effectively this is a circumstance in which the trading of water between periods has reached an equilibrium value equivalent to the economic concept of supply and demand equilibrium. At this equilibrium point the price that a future time period is willing to pay to gain a unit of water is equal to the cost that would be incurred were the current time period to relinquish that unit of water. The MWV in all periods would be equal to the equilibrium 'price' or marginal value at which this equilibrium is achieved.

However the addition of storage or release bounds to the reservoir significantly alters the nature of the produced MWVs. For storage bounds this is because where a unit of water cannot be stored to generate electricity in the future, then the value of storing that water must be 0. There is no economic gain that can be derived from a physically impossible trade. This has implications for the situation where a unit of water is available, and the future value of releasing in a future period is higher than the value of releasing immediately, yet the reservoir cannot hold the unit of water as it has insufficient capacity. In this circumstance the better decision is to release the unit of water immediately and gain the benefit that can be derived through release rather than attempt to store the water and be required to spill it when the reservoir is too full. The value of storing a unit of water from a period can be constrained by the maximum reservoir capacity either in the current period, or any future periods prior to the intended release. Ultimately this means that the trade of units of water between periods to reach a single MWV cannot take place and so the marginal value of water is not constant. This means that the marginal value of storing a unit of water for future release would be 0 even if the marginal value for an additional unit of water for that future period were higher than any other period.

Upper reservoir constraints are discussed above in detail. The lower reservoir constraints however must also be taken into consideration. These limit the ability for water to be drawn from the reservoir beyond the lower reservoir limit for generation. This is usually a constraint representing legal or environmental controls on the reservoir lake. We have assumed that in practice given the releases occur throughout the week and simultaneous to the inflows that there is sufficient flexibility in these bounds to cope with the real intra-period uncertainty about when these inflows actually arrive and are released.

From the perspective of a decision maker, given a particular beginning of period storage level (State 1), inflows will occur (Step 1) to form an intermediate storage state (State 2, DSS') which may exceed reservoir storage bounds. Based on this intermediate storage state (State 2) and a known function relating release quantities with marginal value a release decision must be made (Step 2, DSR) to form a new system state (State 3,DSS). This new system state will then be subject to reservoir storage constraints in the inter-period problem. However, in our theoretical CDDP hydro-thermal scheduling model we use backwards induction. Thus, for a known State 3 (DSS), information on the value of release must be added (Step2, DSR). This forms the interim state (State 2, DSS') from which the impact of inflows is subtracted (F). This forms the beginning of period state (State 1), which is truncated to storage limits in order to form the end of period state (State 3) for the time period before. Hence the DSS for a period is formed as a sum of inflow adjusted DSRs for later time periods, truncated so as to lie within feasible limits. Bellman's principle of optimality applies, if we assume that inflows, and benefits, are independent between periods. So the DSS for any period can be formed from the DSR for the following period, the inflow from the following period and the DSS for the following period.

8.1 Filling the Load Duration Curve

A primary aspect of reservoir management in the electricity sector is determining the basis on which the optimal release decision described above should be made: this involves forming the DSR. To provide context for this determination we must consider the context of the wider electricity sector. In this sector demand for electricity must be met from a variety of generation sources, with different costs. The total demand varies over time. For a given time interval this variation can be represented by a Load Duration Curve (LDC) which represents the percentage of time that the demand for electricity is expected to exceed any particular load level within the interval.

We assume that this demand is met by 'filling the LDC' in 'merit order'. That is, the load requirements implied by the LDC is met by the least expensive (lowest short run marginal cost) generation source first, as shown in the Figure 12 below. The height of each generation ‘slice’ indicates the quantity of instantaneous generation capacity that the facility is providing at that SRMC in GW. This means that the height of a ‘slice’ is generally constrained by the facility’s generation capacity. The width of each ‘slice’ corresponds to the percentage of time that capacity is required. The total area of each generation facility ‘slice’ corresponds to the total quantity of energy supplied over that time period in GWh. In the case of hydro-generation this total area will also correspond to the total quantity of energy equivalent water released from the reservoir. Cheaper sources of generation generate a larger percentage of the time than more expensive generators. Generators that generate a large percentage of the time are known as filling the base load, whereas those only generating for a small percentage are known as filling the peak load. The reality of dispatching generators to meet load is more complex due to network constraints, losses and reserve requirements. However we will assume a merit order based on SRMC is an adequate representation for the purpose of this discussion. For the purposes of this thesis, a station is described as being the highest in the merit order when it has the lowest SRMC of any generation station, conversely a generation facility is described as the lowest when it has the highest SRMC. The generation facility with the highest SRMC in the complete merit order usually represents the option of not meeting that portion of the load. The cost associated with this theoretical generation facility is the cost of shortage.

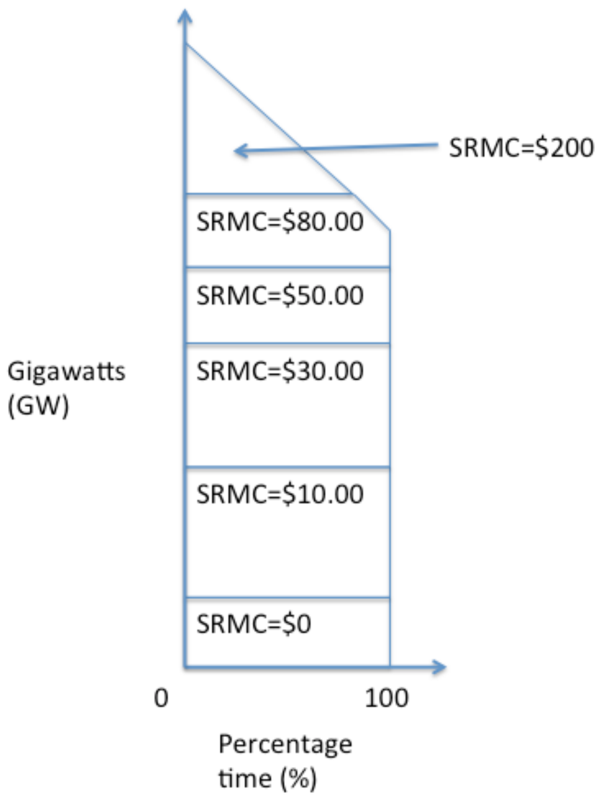


Figure 12: Filled LDC with nominal Short Run Marginal Costs (SRMCs)

We assume that the LDC is filled in merit order based on SRMC, however not all fuel sources have an explicit associated cost. These generation sources include wind generation and hydro-electric generation. Wind generation is intermittent; its availability is based on whether or not the wind is blowing in the right direction at the right speed at any particular time. Consequently when wind generation is available it is a resource that must either be used or lost and so has an implicit SRMC of 0. This means where wind is available it will be dispatched. Hydro-electric generation likewise has no explicit SRMC, however, where storage is available some value can be derived from storing the water for later use. Hence, determining the portion of the load to be met by hydro-electric generation is more difficult.

Water is a resource with no explicit cost so the cost associated with hydroelectric generation is the opportunity cost of being unable to use the water to replace thermal generation in the current time period to replace another thermal generator, or avoid shortage. Where there is no storage capacity, water cannot be used for later generation and so this MWV will always the SRMC of the thermal generation that would be avoided if water is used for generation rather than spilt. Figure 13 illustrates how release from Hydro 1 will decrease when the MWV of Hydro 1 rises above the marginal cost of a particular thermal generator. Clearly, each generators marginal cost is a critical MWV, at which the quantity of water released may change. At the critical MWW, any combination of the two sources that produce the same total generation has the same cost, and any intermediate release level can be optimal. Thus, increasing MWV from zero to infinity so that reservoir release starts at the bottom of the LDC and moves up to the top of the LDC, produces a monotone decreasing DSR, consisting of a series of quantity steps. When a merit order swap occurs at a MWV although the amount Hydro 1 is generating changes while Hydro 1 has a MWV equal to the SRMC of the generator that it is swapping places with the total cost of meeting this period's demand remains the same.

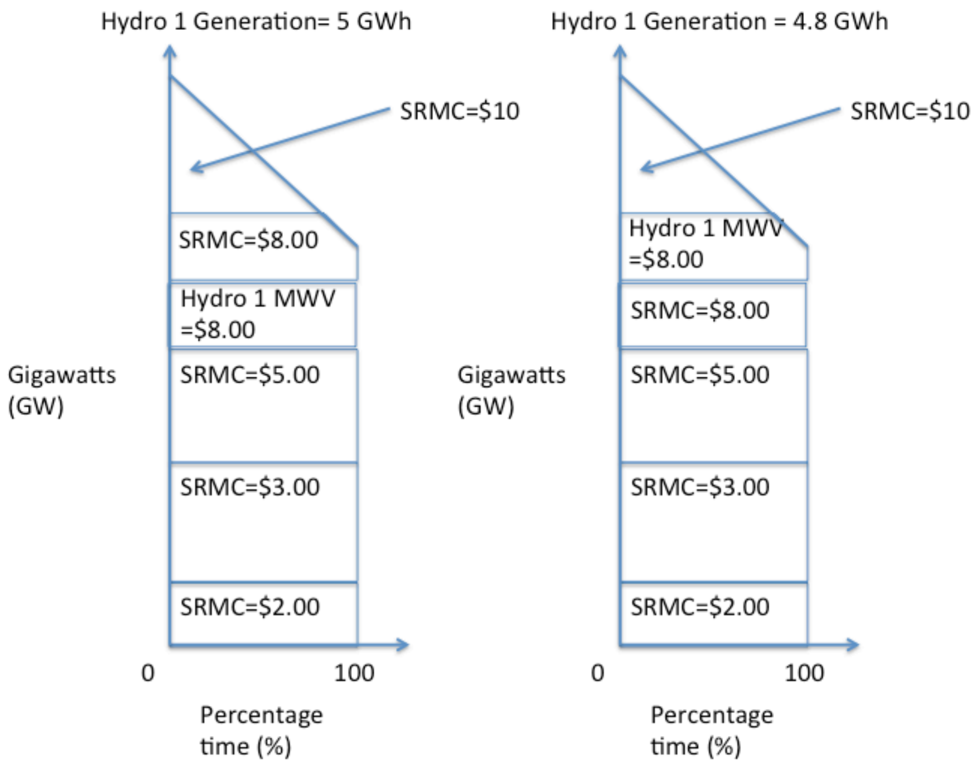


Figure 13: Hydro 1 is swapped in the merit order for a generator with a SRMC of $8.00

However, the release from Hydro 1 will remain constant provided that it is not involved in another swap in the merit order. This means that the generation level in Figure 13 above will remain constant for any MWV of Hydro 1 between $8.00 and $10.00 inclusive. This is because it will retain the same position in the merit order until a further swap occurs. This is shown in Figure 14 below.

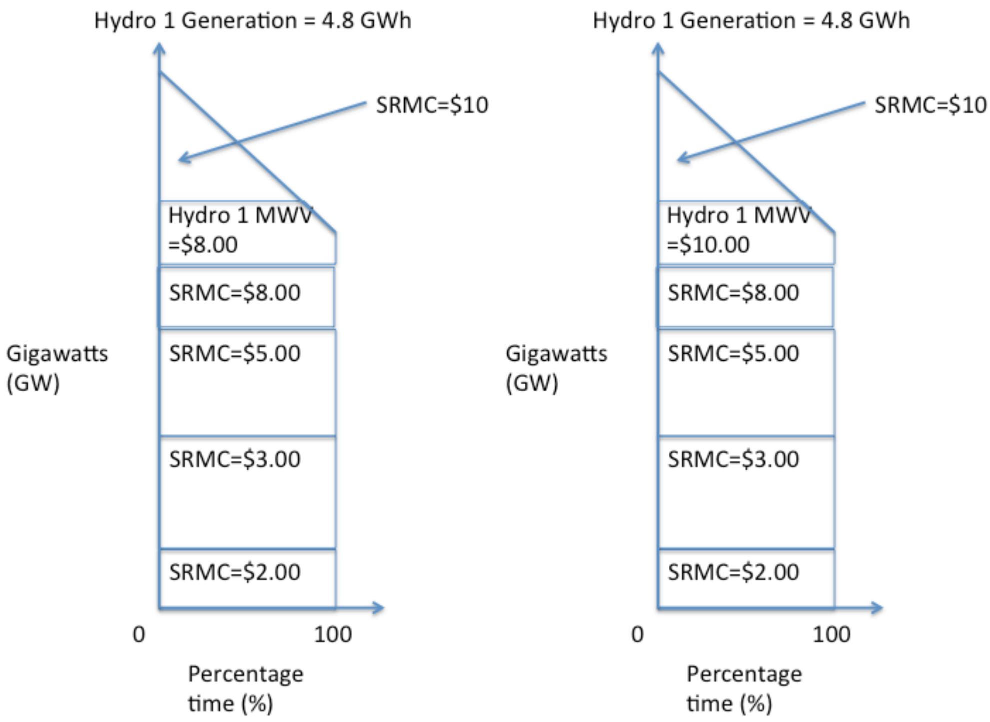


Figure 14: Hydro 1's MWV increases, no merit order swap occurs

Release bounds are implicitly included in the filling of the LDC. The release bounds correspond to the limit on the height of the hydroelectric 'slice' in filling the LDC. To form the DSR from the filled LDC the first MWV for which the release is calculated is where the MWV is so low that the hydro facility is at the top of the merit order.. The release from the reservoir corresponds to the area of the 'slice' of load the hydro generator fills given the maximum instantaneous capacity the facility and the number of hours that instantaneous capacity is supplied. The number of hours supplied is equal to the percentage of time multiplied by the hours in the time period.

From this MWV the next MWV for which the DSR is filled is the lowest SRMC. At this SRMC the maximum release level is equal to the minimum release level for the first MWV lower than this MWV for filling the DSR. This is the equivalent of the MWV changing but the placing in the merit order has not changed as shown by Figure 14 above. The minimum release associated with that MWV is where the hydro reservoir has been completely displaced in the merit order of filling the LDC. This scenario is shown in Figure 13 above. Given this new position in the merit order the release from the reservoir again corresponds to the area of the 'slice' of load the hydro generator now fills. This process continues as the hydro facility is displaced by each alternative generation source in turn and the SRMC for that generation source is recorded alongside the minimum reservoir release where the reservoir MWV is equal to that SRMC. In general it is only necessary to calculate the minimum release as for each of these MWVs the maximum release is delineated by the minimum release of the reservoir for the next MWV up in the merit order. The exception to this is the lowest MWV, however for this MWV the upper release is delineated by the case where the hydro reservoir is top of the merit order.

8.2 Our Specific Case

An approximation of the MWVs for each unit of water given a particular storage level can be determined by a long term optimisation model, in our case CDDP is used. CDDP takes all feasible future opportunities into account in determining the MWV (J. G. Culy, Read, Wright, & New Zealand Institute of Economic, 1995). Correspondingly CDDP produces guidelines for release and storage based on these MWVs. However, we note that CDDP is a deterministic model and so the influence of ever-changing capacity, inflow and load circumstances on the MWVs will be the subject of further discussion under the SCDDP section below (see page 51).

For the final time period, we assume a known DSS represents the MWV of water stored beyond the planning horizon. CDDP is then solved using backwards induction as described in the section on CDDP above (see page 28). The general algorithm is stated on the left of Figure 15 below. Figure 15 is a diagram representing the impact in the single reservoir problem likewise illustrates each step. These steps are numbered such that they correlate to Figure 9 above. Step 2 shows the interim DSS' for the current period being formed by adding the DSR for this period, to the known DSS (State3) already formed from the next period. In Step 1 the DSS’ is adjusted for the expected inflow, and the result is the DSS for the previous period before reservoir limits are applied (State 1). The inter-period truncation forms the DSS (State 3) for the end of the previous period. For the final time period the known DSS is representative of the expected value of storage beyond the modelled horizon we have assumed this is equal to the DSR for the first time period.

$DSS_{t+1} = s_{t+1}(v) \quad \forall v$

$DSR_t = r_t(v) \quad \forall v$

$DSS'_t = u_t(v) = s_{t+1}(v) + r_t(v) \quad \forall v$

$DSS_t = u_t(v) - F \quad \forall v$

$$\begin{aligned} DSS_t \\ &= \max\,(u_t(v) - F, Lower\ Res\ Limit) \\ &= \min\,(u_t(v) - F, Upper\ Res\ Limit) \\ \forall v \end{aligned}$$

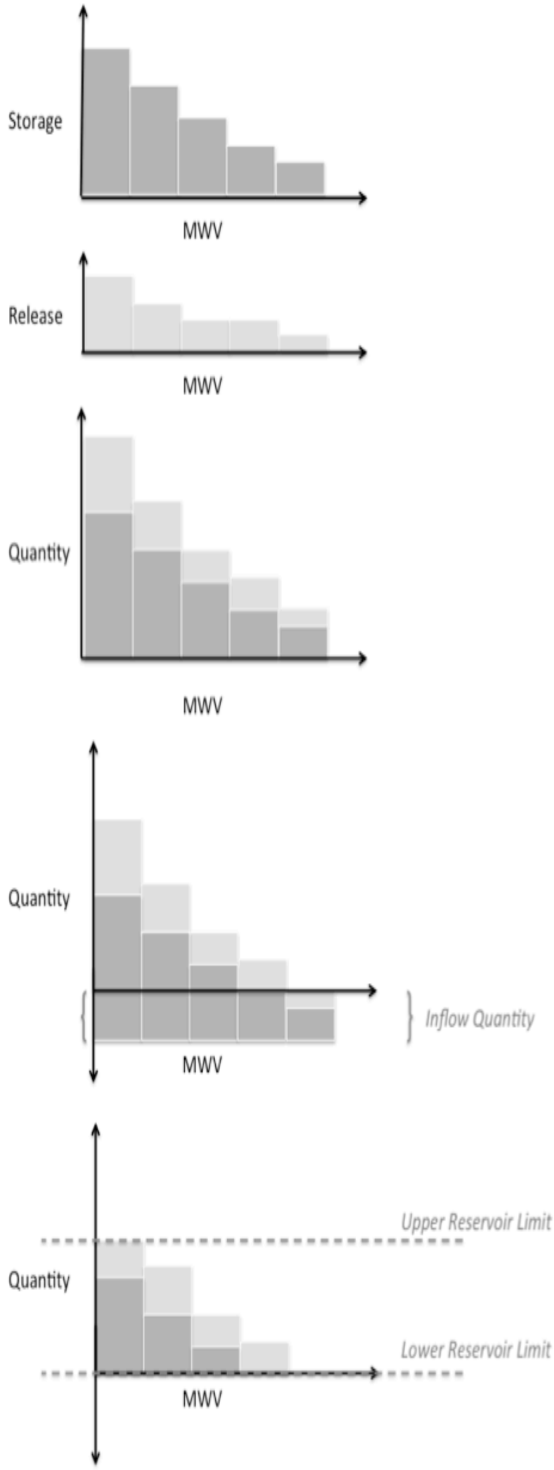


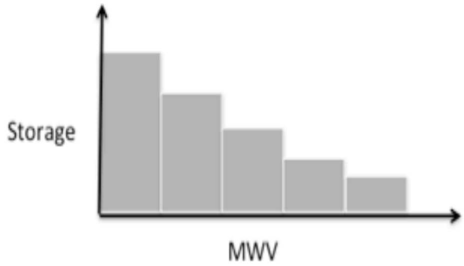
Figure 15: CDDP algorithm for forming DSS'

Essentially this same process applies in higher dimensions, for n n-dimensional DSSs. Each of these representing the MWV of water stored in each reservoir, as a function of all reservoir levels. The computation required for each point on these surfaces is trivial, but the key source of computational burden is the number of points required to represent the surfaces. The most elementary approach would be to store, and update, n MWVs for every point in an n-dimensional grid. Using an arbitrary grid to store only a subset of these points could reduce the computational burden this representation implies. However, some of the complexity of the surface could be lost. In response to this the algorithms described here only store and update the storage vector corresponding to a limited set of critical MWV levels, and MWV ratios. SPECTRA, for example, formed North Island storage guidelines, along which the North Island reservoir MWV was equal to critical North Island marginal cost levels defined over an arbitrary grid of South Island storage levels. SPECTRA also included import and export transfer guidelines along which the South Island MWV equalled the North Island MWV +/- marginal HVDC losses. Another representation is that in (Dye & Read, 2012), where MWVs are associated with a fine grid of regularly sized discrete water quantities. These both, however, would tend to limit generalisability to higher reservoir numbers. The former is limited due to issues with visualization in higher dimensions. The latter is limited due to the implicit computational burden.

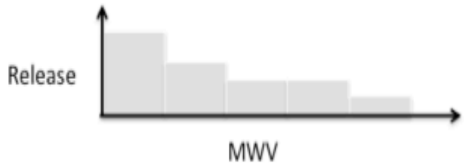
In order to ensure that our algorithm is more readily generalisable to more reservoirs we form a set of critical price vectors, these represent the critical MWV levels and ratios that may occur over the planning horizon. Using only these key critical points reduces the computational burden and improves the prospects of developing a consistent method for expansion to more reservoirs. As discussed above, these critical MWVs are sufficient to represent the demand surfaces, as the detail of the MWV surfaces between these points does not need to be recorded. This is because these critical levels and ratios are the only points at which the release solution changes. This is shown in Figure 13 and Figure 14 above. It is these SRMCs that are the basis of the MWVs, given that we are ignoring wastage, holding costs and discounting, no other MWV levels can actually occur in this deterministic case. Each unit of water will eventually be used to satisfy demand, and replace generation at one of these marginal cost levels. This means that CDDP does not determine which level that fulfilment will be at but instead the deterministic CDDP algorithm determines minimum and maximum storage and release values for which each possible MWV level applies. And to form the DSS CDDP will only determine the storage pair corresponding to each critical price pair, recursively, for each period. And so the DSS, DSR and DSS' are all Marginal Water Value Surfaces (MWVSs) that represent water to be stored or released over a discrete set of MWVs. The algorithms above become:

Where V is the set of all critical MWV levels.

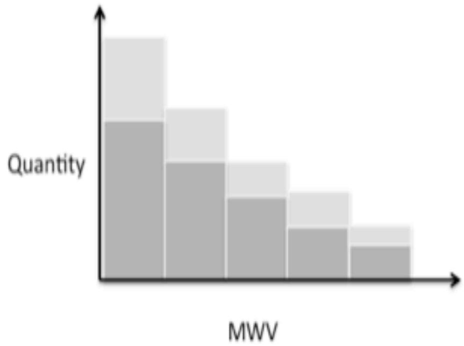
$$DSS_{t+1} = S_{t+1}(v) \quad \forall v \in V$$



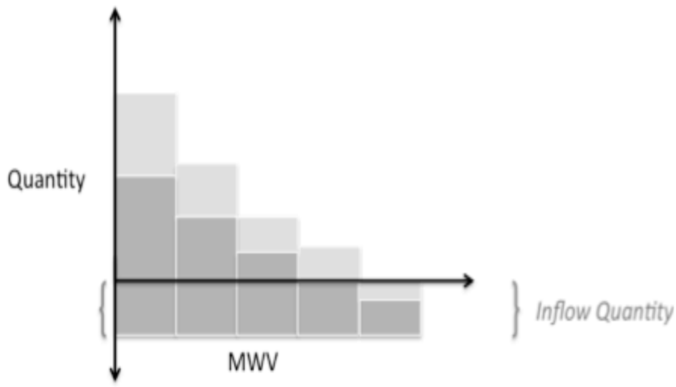
$$DSR_t = r_t(v) \quad \forall v \in V$$



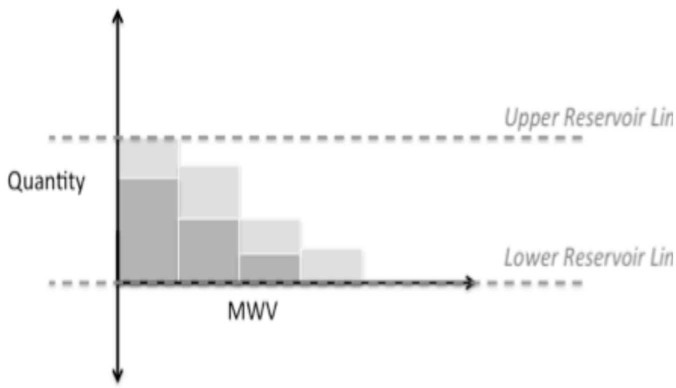
$$DSS'_t = u_t(v) = s_{t+1}(v) + r_t(v) \quad \forall v \in V$$



$$DSS_t = u_t(v) - F \quad \forall v \in V$$



$$DSS_t = \max(u_t(v) - F, Lower\ Res\ Limit)$$
$$= min(u_t(v) - F, Upper\ Res\ Limit)$$
$$\forall v \in V$$



This methodology also allows all DSRs to be pre-computed into the critical MWV format that will determine their use in the algorithm. This simplifies the inter-period process considerably. Likewise through representing all reservoir storage and inflow in comparable energy terms, the necessity of conversions within the CDDP algorithm itself is eliminated.

We have generalized a stochastic extension from this approach to deal with a range of different New Zealand based one and two reservoir cases. The two-reservoir problem and the stochastic variations on that will be discussed in further depth in the following sections. Although higher reservoir problems are the underlying motivation for the development of this algorithm, here we will first discuss a generic single reservoir problem.

9. Stochastic Constructive Dual Dynamic Programming

9.1 Original Formulation

Dynamic Programming (DP) and variants thereof are particularly valued in their ability to readily take into consideration stochasticity and CDDP is no exception to this. DP problems are by nature neatly arranged into sub-problems. This means that from a structural perspective, the uncertainty associated with the outcome from each decision can be taken into account separately for each sub-problem. Where there are separable problems then separable uncertain outcomes can be intuitively incorporated into the problem formulations. For the purposes of this thesis, we assume that the probability of an uncertain event in a given time period is independent from the resolution of uncertainty in any other time period. However, we note that the application of dependencies in uncertainty is explored for DP in Yang (1995) and is likewise currently being developed for SCDDP at the Univesity of Canterbury. We have focused on independent stochastic outcomes as incorporating such stochasticity is relatively intuitive, implies a lower computation burden and is a necessary step in developing the theory before considering the extension to correlated stochastic outcomes.

The addition of stochastic inflows means that there is increased uncertainty as to the ultimate value of stored water. As a storage decision is effectively a decision to release that unit of water in a particular future period then each inter-period inflow uncertainty between the initial time of storage and the time of intended release will increase the uncertainty of the system state at the intended time of release. Correspondingly the stochastic inflows mean that we are uncertain about the value that will be accrued by releasing that stored unit when the actual inflow occurs. From the perspective of computational burden this uncertainty implies the memory required for recording the many possible future system states. Hence where there are a large number of possible system states then it is likely that the incorporation of these stochastic outcomes in a DP formulation may increase the solve time of any problem beyond feasible limits very rapidly. This issue has likewise been prevalent in

SCDDP implementations. This aspect has severely curtailed the ongoing development of commercial hydrothermal scheduling applications such as SPECTRA to incorporate higher reservoir numbers or detailed system characteristics (Halliburton, 1994).

The core principle that differentiates SCDDP from CDDP is the inclusion of uncertain inflows in the inter-period problem. We assume that from a process perspective that the release decision (Step 2) for the current time period is made after the inflow state is known (Step 1). As described above, in practice the release decisions and inflow discovery will occur concurrently, however the reservoir manager is likely to have an approximate idea of likely inflows based on recent weather patterns and forecasts. The choice of stage boundaries is arbitrary once the order of events has been determined. However, a storage decision is also implicit when the release decision is made (Step 2). The storage decision is made based on a known end-of period state (State 3) and some expectation of possible inflows in the following time period. Specifically it is based on the expected marginal value for possible storage levels. The realization of the inflow state for the next period is then the first event in that period and a release and storage decision can then be made on that basis. Figure 16 below is and expansion of Figure 9 to incorporate stochastic inflows.

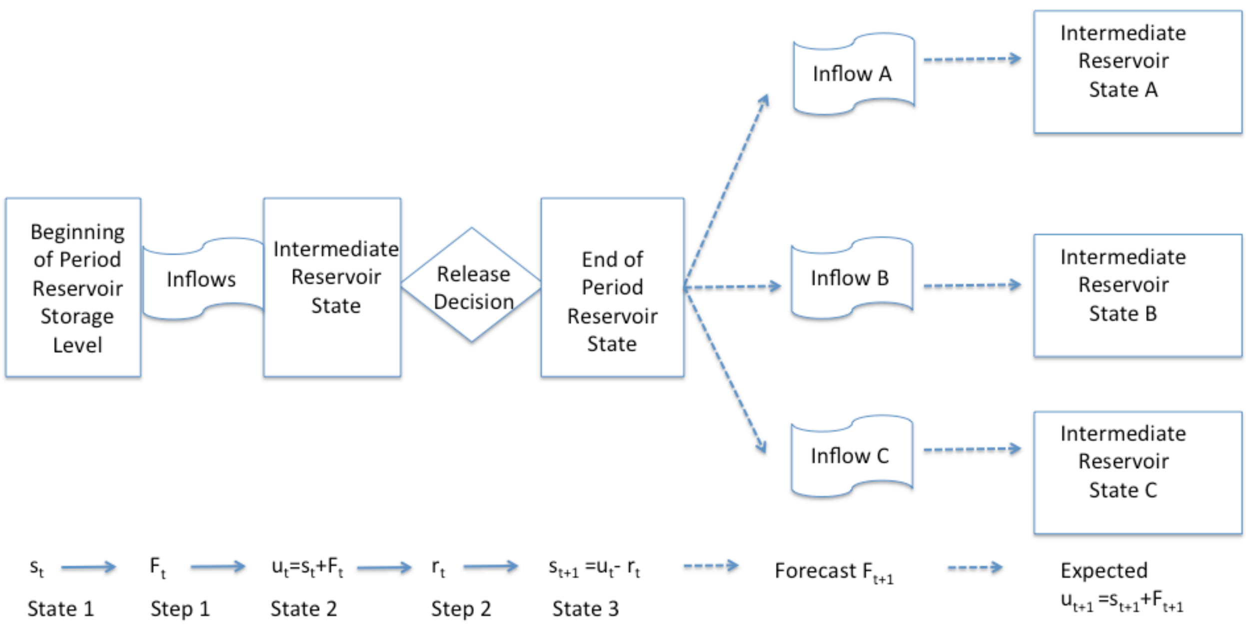


Figure 16: Single period reservoir management problem with uncertain inflows

If we consider the diagram above in terms of the process of backwards induction this implies that where the demand surface for release from a given period is known, and the intermediate system state for the next period is known then to fully capture all of the possible system states given the inflow uncertainty the problem would be as shown in Figure 17 below.

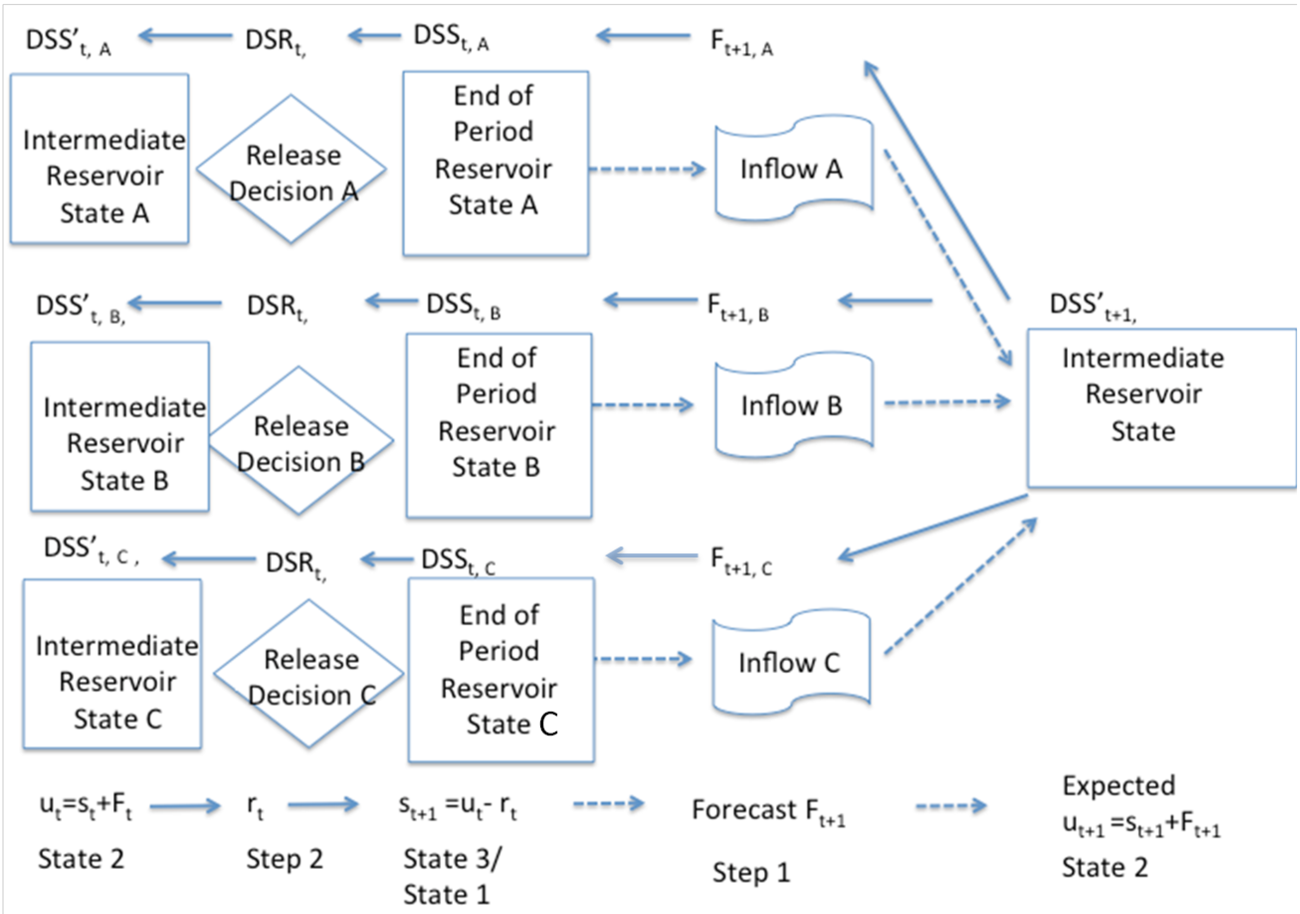


Figure 17: Stochasticity in the backwards induction single reservoir problem

However, to capture this level of detail each of the 3 resulting Intermediate Reservoir State's for time period t would likewise separately result in a tree structure as large as the above for the time period before. Self-evidently this tree structure would become unmanageably large even with a simple representation of stochasticity as the number of time periods increased. Consequently in implementing SCDDP we have chosen to merge the end of period states to develop a version of State 1/3. This is a demand surface corresponds to a DSS based on the expected marginal value of storage over all uncertain future states. This new State 2 will be described as the Expected Demand Surface for Storage (EDSS). On the basis of the Expected Demand Surface for Storage (EDSS) the storage and release decisions can be made based known inflows for this period and an expected marginal value of storage for later periods. These will together form the expected intermediate reservoir state. This is shown in Figure 18 below.

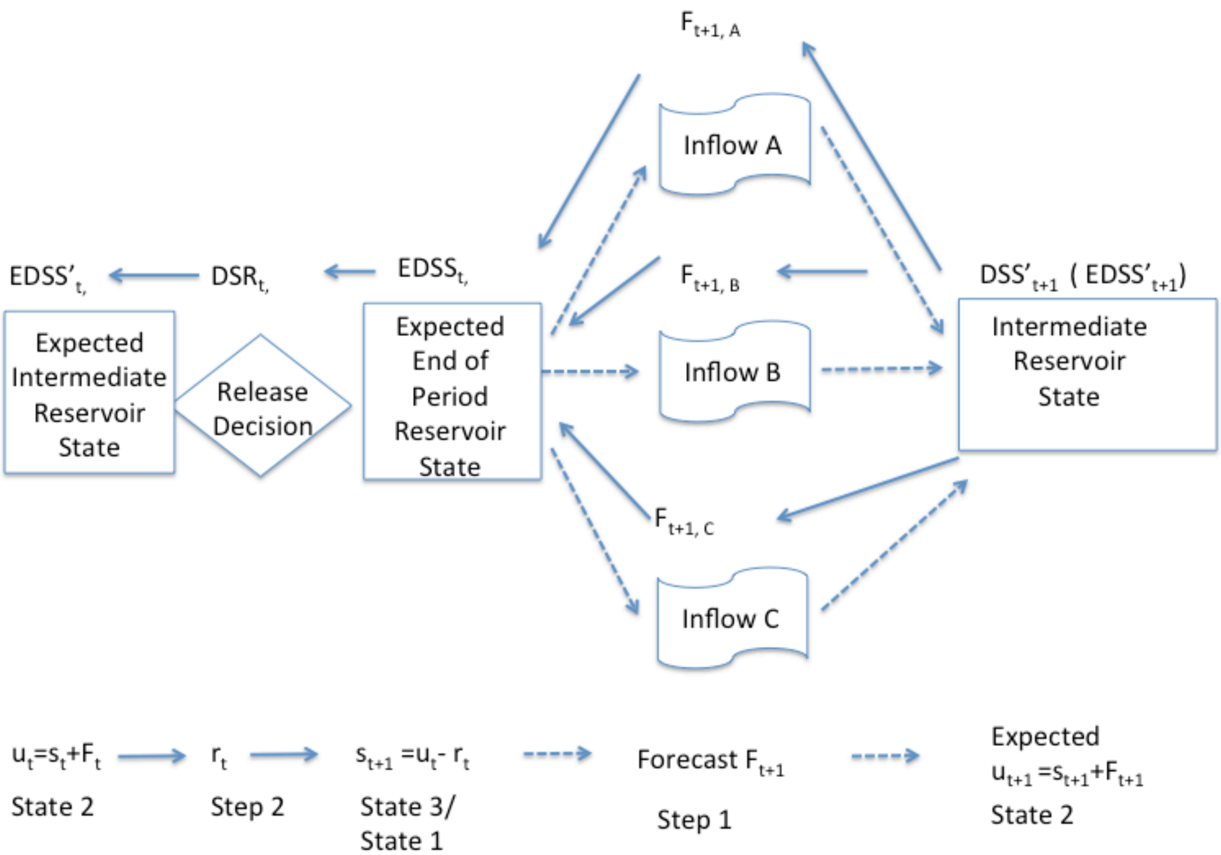


Figure 18: SCDDP applied to single period reservoir management problem

State 3 for time period t is the equivalent of State 1 for time period t+1. This state is the EDSS and is created by the addition of the probability weighted future possible State 2 system states, or the expected intermediate reservoir state for the next time period. As discussed for CDDP above this intermediate reservoir state is effectively a combination of the expected value for release in all later time periods. In practice this is the sum of the probability weighted DSRs for each future time period. Since future inflows are uncertain we cannot predict with certainty to what extent future inflows will be able to fulfil future demand and hence the value of storage for release as part of each future DSR is unclear. Where the future inflows are higher than the expected mean inflow then the actual value derived from the stored water will be less than the expected value. Conversely where the future inflows are lower than the expected mean inflow the actual value derived from the stored water will be higher than the expected value. Where inflows are stochastic the expected marginal water value (EMWV) as represented in the EDSS can also be affected by the prospect of the reservoir being full due to high inflow levels. This would cause units to spill and so the actual MWV of the stored units that are spilled is 0. The expected MWV can also be affected by lower storage constraints as described in the CDDP section above. Where there is the possibility of spill then the EMWV stored in the EDSS for a water unit will be lowered proportionate to the probability that unit will be spilled. Conversely, where there is the possibility of violating the lower storage constraint then this usually implies that the reservoir is at risk of running dry. In this case a unit may have an extremely high actual MWV as this is the value of keeping some water in the reservoir. This will increase the EMWV of the unit in proportion to the probability that it will be needed. Consequently, particularly near the reservoir bounds, the actual MWV of a unit of water can differ significantly from the EMWV. A difficult aspect of this calculation is that we are calculating the EMWV given a set of possible changes in the storage space and the associated probabilities of those changes occurring. This is difficult when the DSS and DSRs are stored as a representation that tracks water volume based on MWV. This SCDDP implementation specifies the range of storage levels that are associated with a particular MWV. In order to calculate an expected MWV each of these storage levels must be adjusted by each potential inflow separately. The portion of the expected MWV for each resulting storage level is calculated by multiplying the probability of that inflow by the MWV the initial storage level would be associated with. In order to construct the expected MWV for a given quantity then each ‘MWV portion’ associated with that water volume level is summed. Hence in order to add the functions of water volume based on MWV then the function for inflow adjusted water volume based on a ‘MWV portion’ must be created. Then these functions are inverted and added together to find a function of expected MWV based on the water volume storage. The resulting function is then inverted again so the SCDDP algorithm can continue to be calculated in the dual space.

In commercially implemented SCDDP problems, the stochastic distribution implemented has typically been discrete. This means that for each sub-problem there are a discrete number of uncertain inflow outcomes in the primal space, and each of these outcomes is associated with a probability-weighted outcome in the dual space. A case where there are 3 different inflow possibilities applied to a single reservoir MWV curve can be seen below. Note that these inflows shift the MWV curve in the quantity direction (vertically), and then each of these quantities is associated with a probability-weighted price (the probability weights act horizontally). This is because the uncertainty that is naturally resolved is which inflow we are subject to. That resolved shift then implies a MWV, this MWV will be used to form our EDSS.

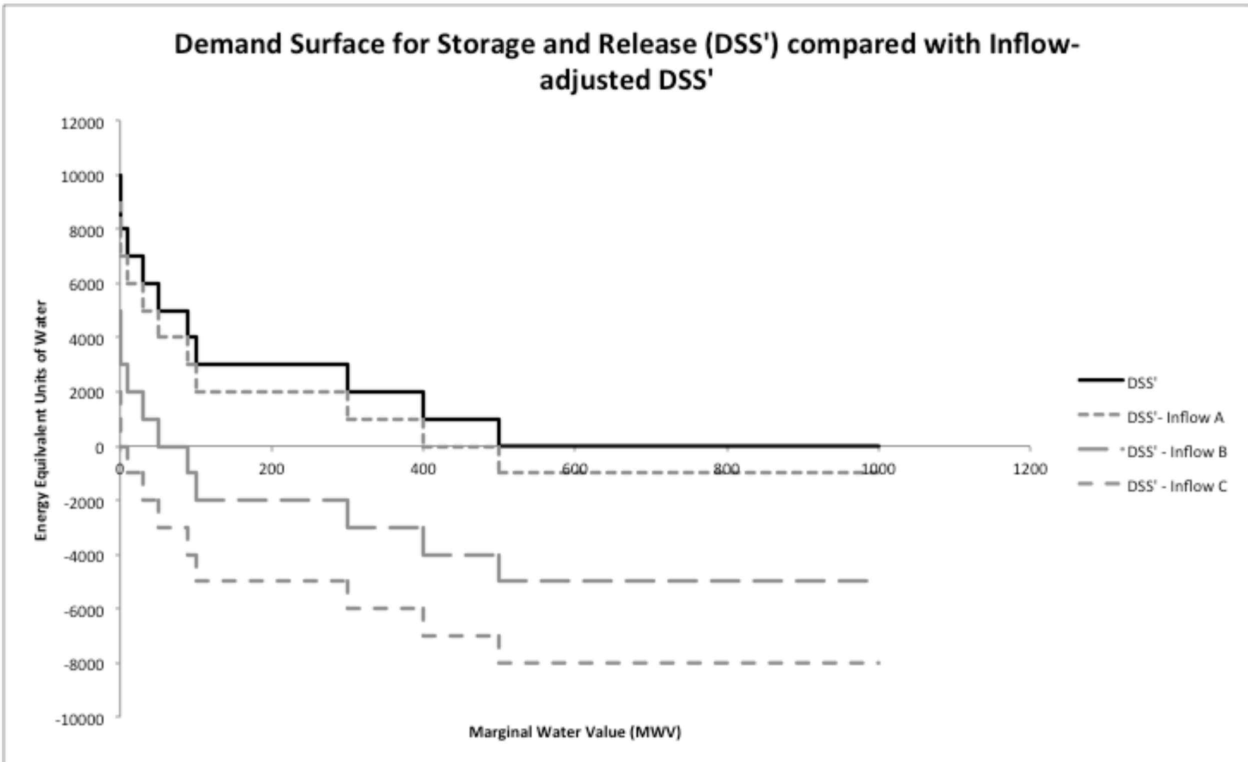


Figure 19: Inflow Adjusted Demand Surface for Storage and Release

Figure 19, above, shows that where there are multiple possible outcomes, then likewise there are multiple possible prices for the single storage level.[2] To find the expected MWV for a given storage level, the probability weighted MWV surfaces

[2] The data used in these figures is representative only as the real data is too granular to readily reflect the impact of these changes on the corner components of the Key Surfaces.

must be added in the price dimension, for every relevant storage level. The formation of the EDSS from 7.1 Key Surfaces is shown in Figure 20 below.

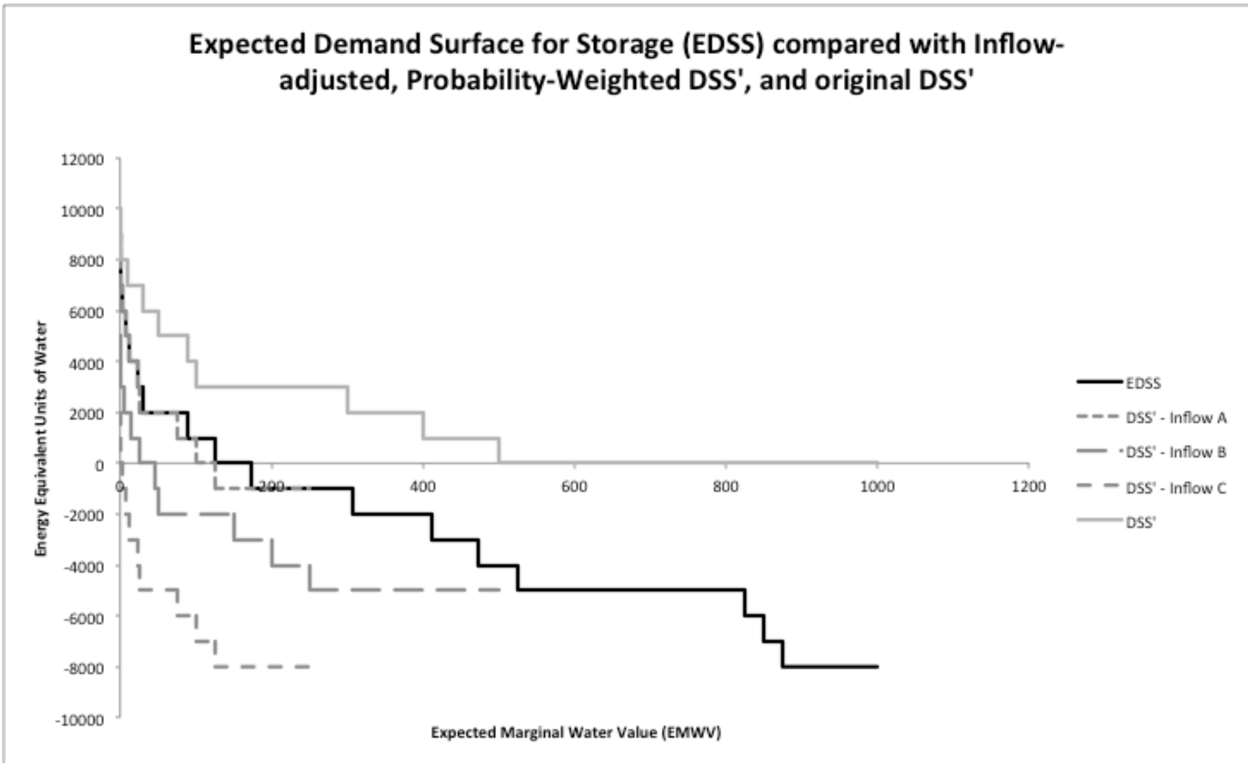


Figure 20: EDSS formation from Inflow-adjusted Probability-Weighted DSS' components

The addition required to form an EMWV for each arbitrarily discretised storage level in the reservoir is a computationally trivial exercise. However, where storage levels were discretised to adequately represent the EDSS then performing the addition for every feasible storage level would rapidly become computationally intensive as the quantity of reservoirs increased. Consequently it is more computationally efficient to only record the impact at a number of key points. These points are where corner-points occur in the EDSS depicted in Figure 20 above.

The use of critical points mirrors the use of the algorithm described for CDDP above. The EMWVs comprising the DSS are formed by adding the probability weighted MWVs . In order to accurately capture the entire DSS compactly then only the EMWV for every water level that an inflow scenario identifies as a critical water level must be formed. This can be seen in Figure 20 above.

This representation is useful for a single period: there are a small number of discrete points that represent the MWVS. However for each additional inflow scenario the quantity of critical points necessary to represent the EDSS is increased. Each inflow scenario results in an increase equal to the number of points used to represent the initial MWVS. The quantity of critical points will then be further increased to include reservoir storage levels corresponding to the critical MWVs of the DSR in order to form the new EDSS'. This is shown in Figure 22 below.

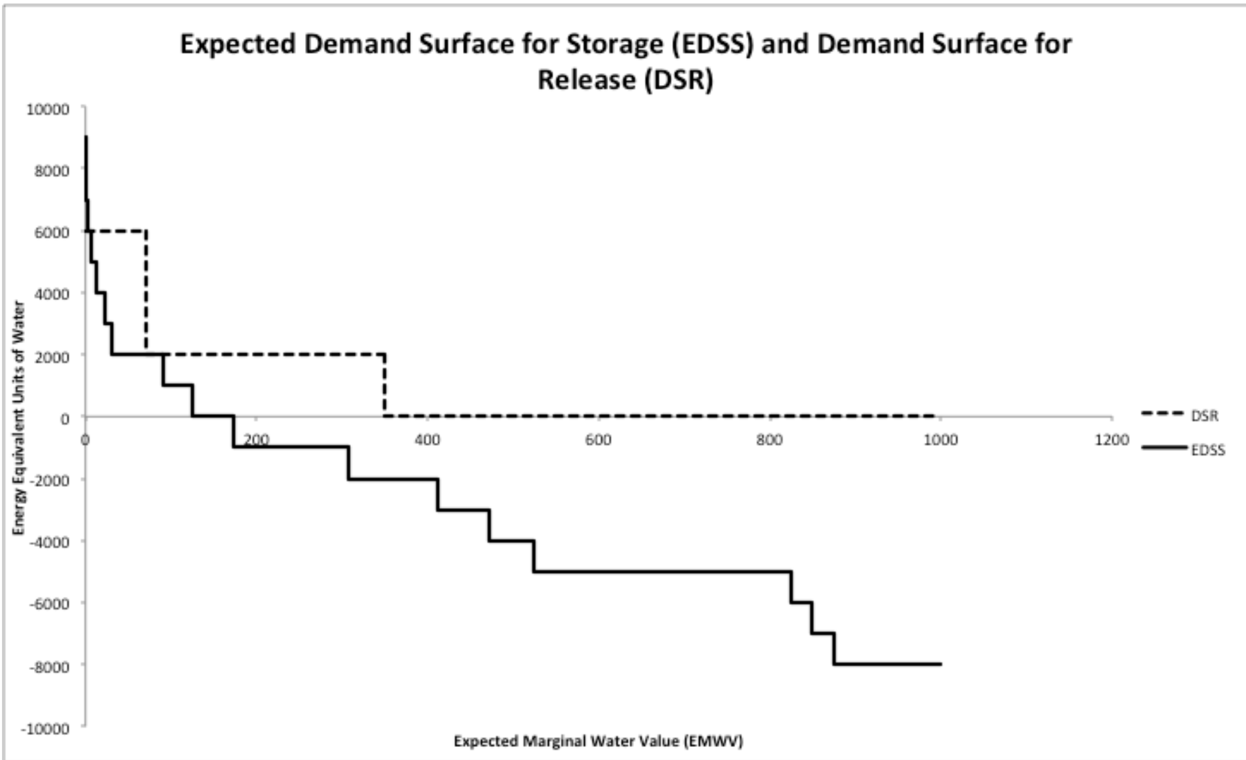


Figure 21: EDSS and DSR

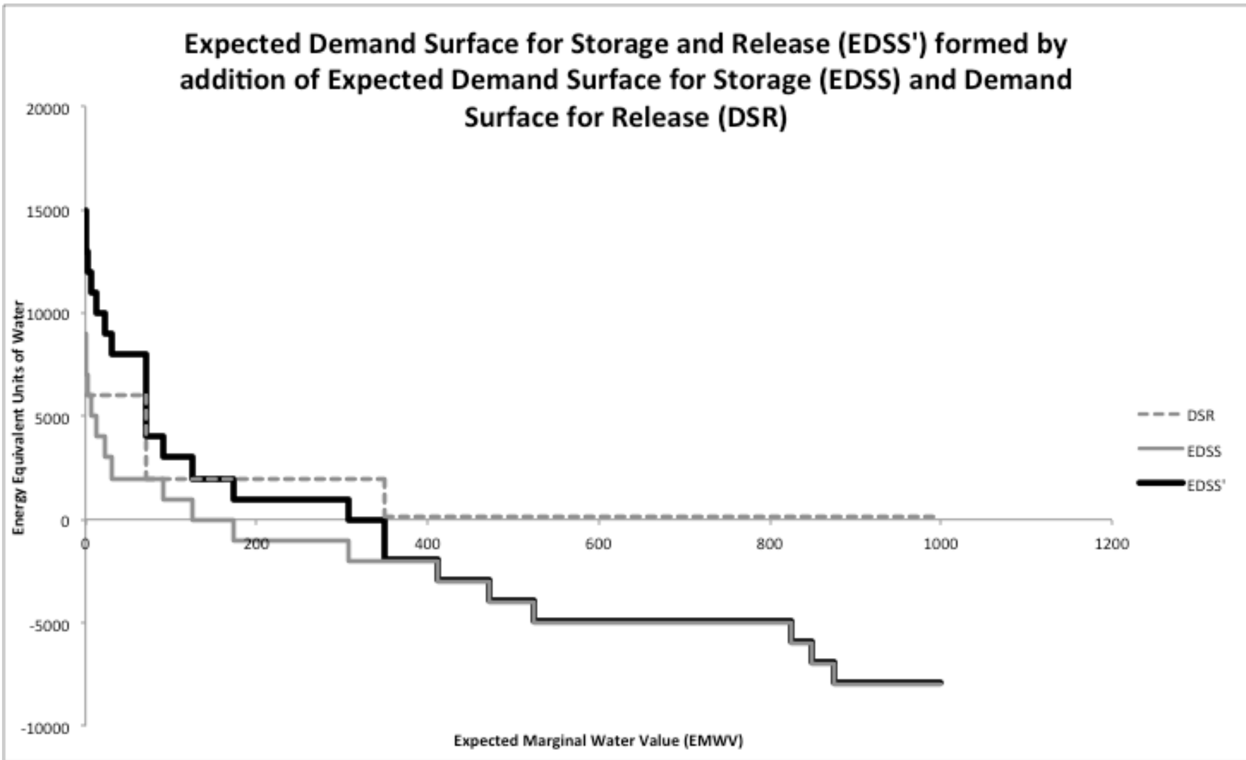


Figure 22: EDSS' formed from addition of EDSS and DSR

When this EDSS' is used as the basis for forming a new EDSS, again the quantity of points will increase based on the number of inflow scenarios and the number of points in the EDSS' representation. This is shown in Figure 24. These increases would rapidly cause computational infeasibility for problems with higher dimensionality, as this would again increase the quantity of points required to take each of these MWVSs into account exponentially. It is in response to this limitation, that further simplification is required.

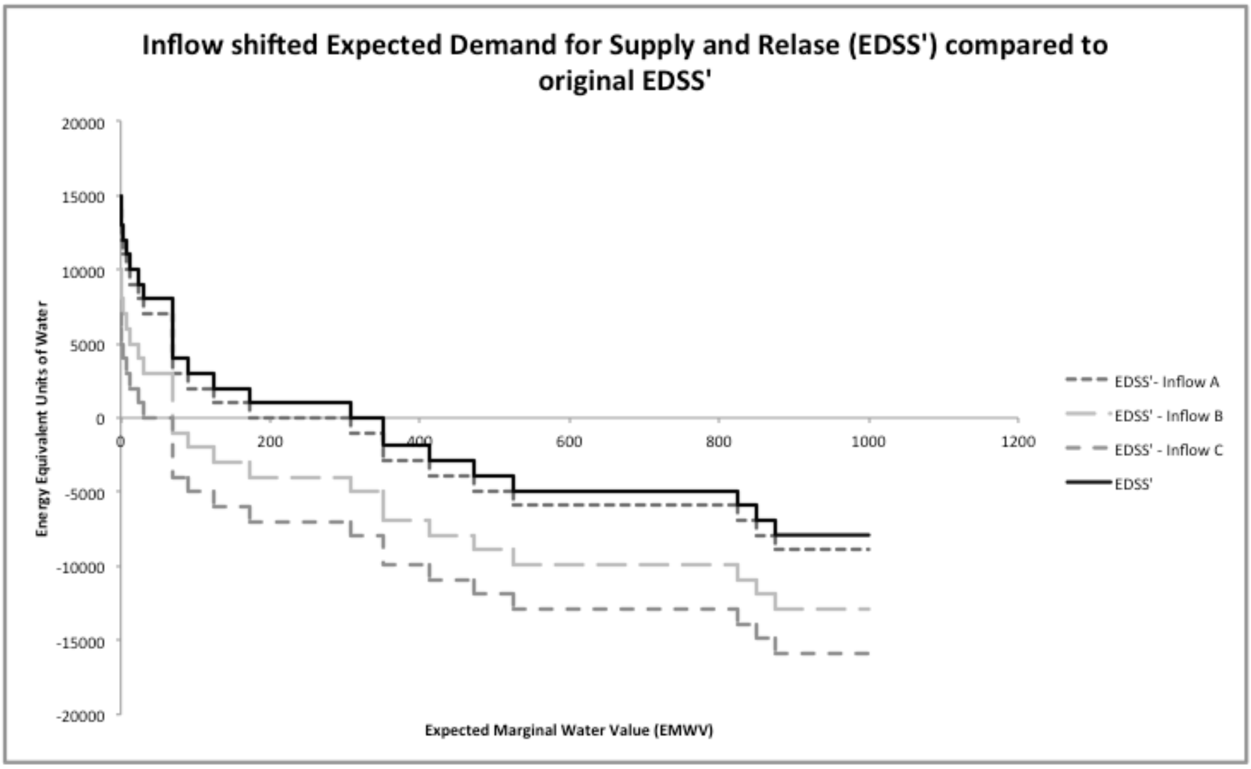


Figure 23: Inflow adjustment to EDSS'

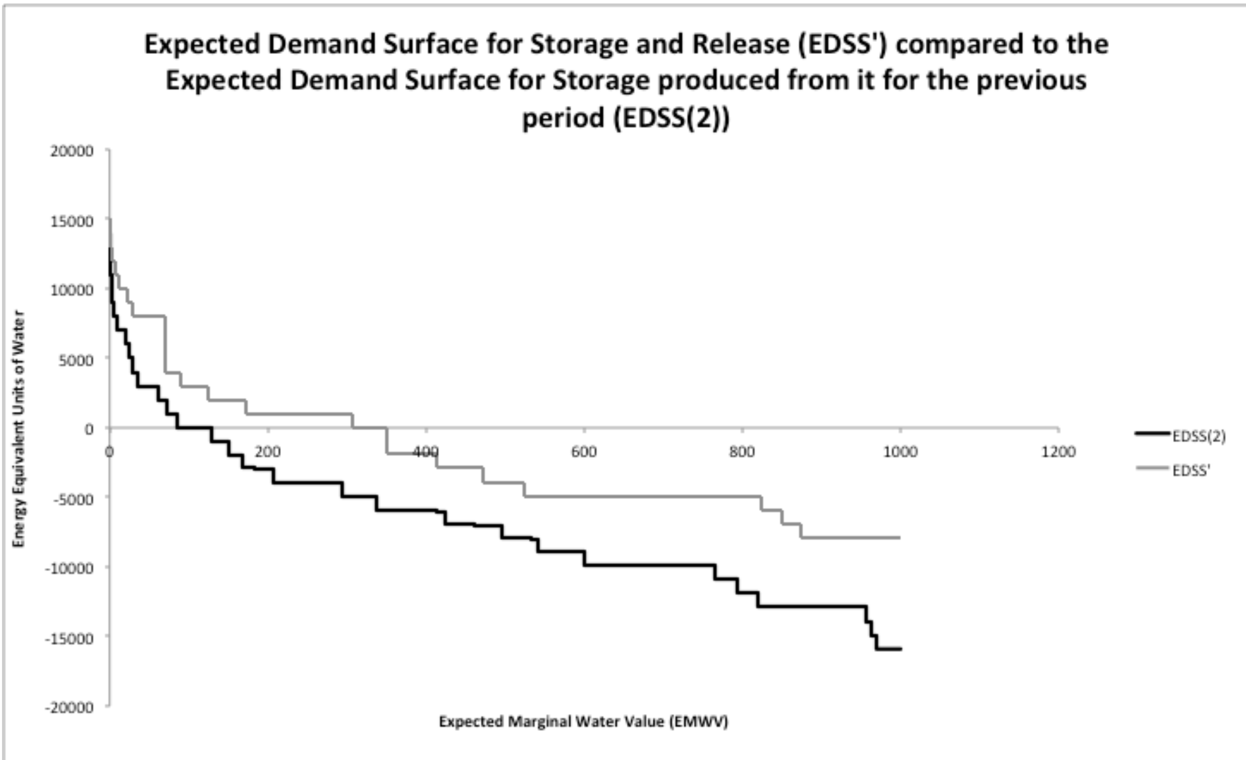


Figure 24: Comparison of the EDSS from the previous period with the EDSS' it is formed from

With sufficiently high numbers of time periods and possible inflows, it becomes evident that the inclusion of uncertainty results in such a large quantity of piecewise constant steps. These steps are numerous and often represent small increments. The function that develops is approximately piecewise linear in appearance. By taking this appearance into account it can be concluded that a simpler approximation would be developed by representing the function as piecewise linear rather than piecewise constant demand surfaces. In response to this the original implementations of SCDDP would approximate this piecewise linear curve by cumulating probability adjusted MWVs for an arbitrary grid of storage points. Notably the small steps seen below are a result of using a discrete approximation for the uncertainty rather than these being a property inherent to the system.

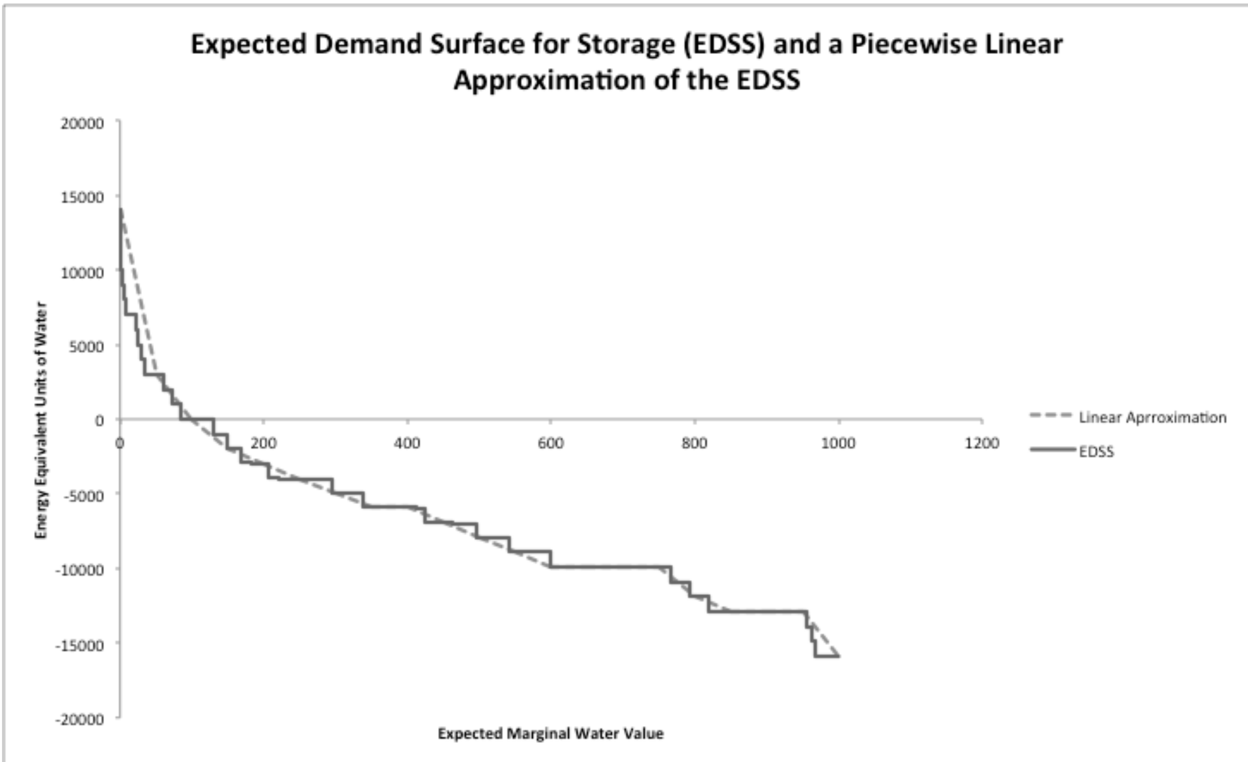


Figure 25: Piecewise linear approximation of the EDSS based on EMWV increments of 50

Discretising the MWVS along the lines described above is advantageous as it reduces the continuous surface to a set of discrete points. This provides initial benefits with respect to computational tractability and allows the technique to maintain a fairly basic sequence of operations. However a trade off will occur. A high number of storage points in the grid will increase the accuracy of the representation and the validity of any results, yet this will increase the computational burden significantly. A lower number of storage points in the grid will decrease the computational burden and yet also decrease the accuracy of the representation of the MWVS.

Representing the MWVS through a discrete set of points may result in a somewhat crude approximation of the true MWVS. The initial limitation of this arbitrary grid in the first instance is that significant and detailed sections may be entirely overlooked by the discretisation. This would be the case below in which there exists a significant change in the slope that relates the reservoir price and quantity. In the below representation due to the choice of discrete points this particular section will be approximated by a much simpler line as displayed in Figure 26 below. The quantity available for one set of prices is evidently considerably over-estimated until the EMWV is equal to 300. Conversely the quantity available between where the EMWV is equal to 300 and where the EMWV is equal to 600 is somewhat underestimated.

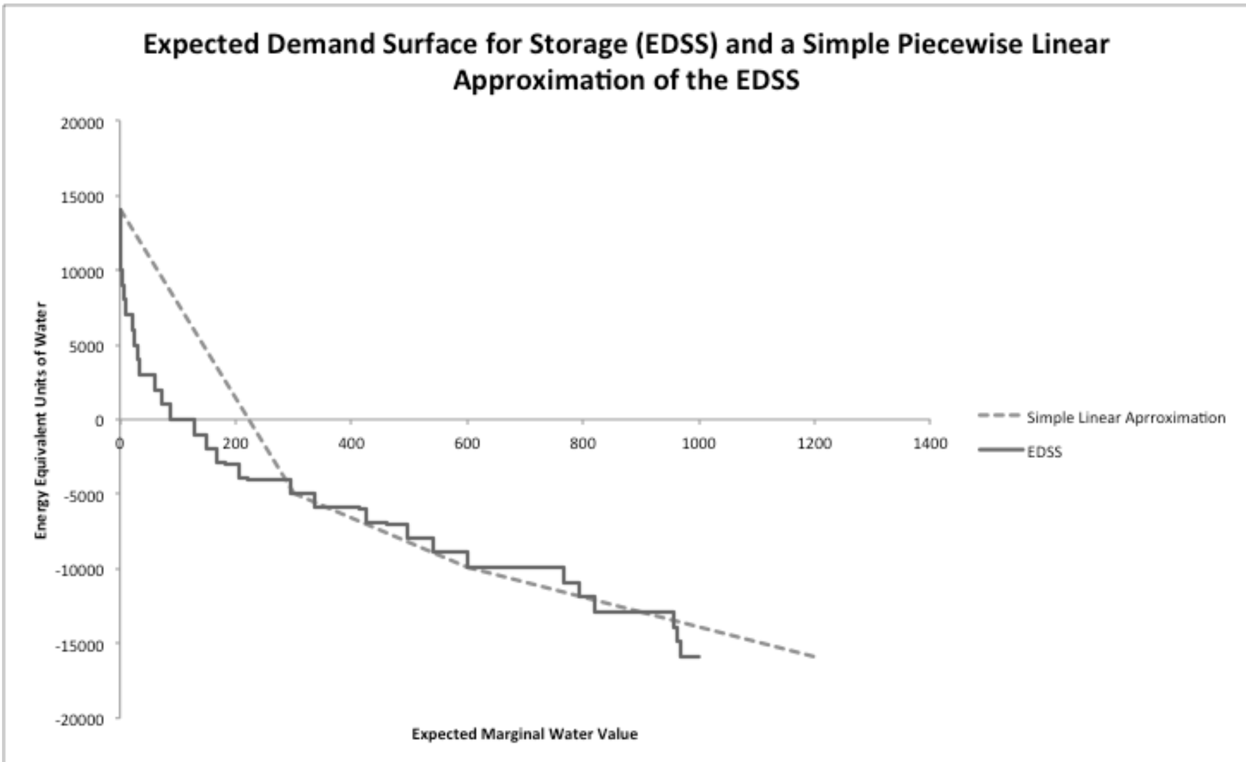


Figure 26: An overly simplistic Piecewise Linear Approximation of the EDSS

Conversely, a small set of discrete points must be used to ensure computational tractability. As the number of reservoirs increases, then for each additional reservoir, the grid is extended into another dimension increasing the problem complexity. The more recorded points, the higher the complexity of the initial problem and hence increases have a greater impact.

9.2 Cornerwise SCDDP Algorithm

In order to limit the number of points which are used to represent the MWVS, the underlying concept of the cornerwise algorithm would imply that the key points need to reflect the cornerwise CDDP structure described above. The cornerwise algorithm can be defined as representing all MWV surfaces in terms of a small subset of points located a small increment either side of the key corner. This allows both the location of the corner and the change in the surface surrounding that corner to be recorded. The nature of the hydro-thermal scheduling problem is such that there are a number of significant MWV points to record. The key points are those in which there is a swap over in the original merit order of filling the LDC. They represent the maximum and minimum releases from the reservoir where there is indifference between the use of reservoir generation and a specific thermal generator at a given MWV. For a single reservoir problem then these key points are also capable of encapsulating the reservoir storage or release limitations. The release quantity only changes at these critical MWVs. The upper reservoir limit can be represented by truncating the EDSS maximum quantity at the first MWV that violates this limit, along with all other storage quantities above this amount. The lower reservoir limit can be represented by truncating the EDSS minimum quantity at the first MWV that violates this limit along with all other storage quantities below this amount. This can be seen on Figure 27 below.

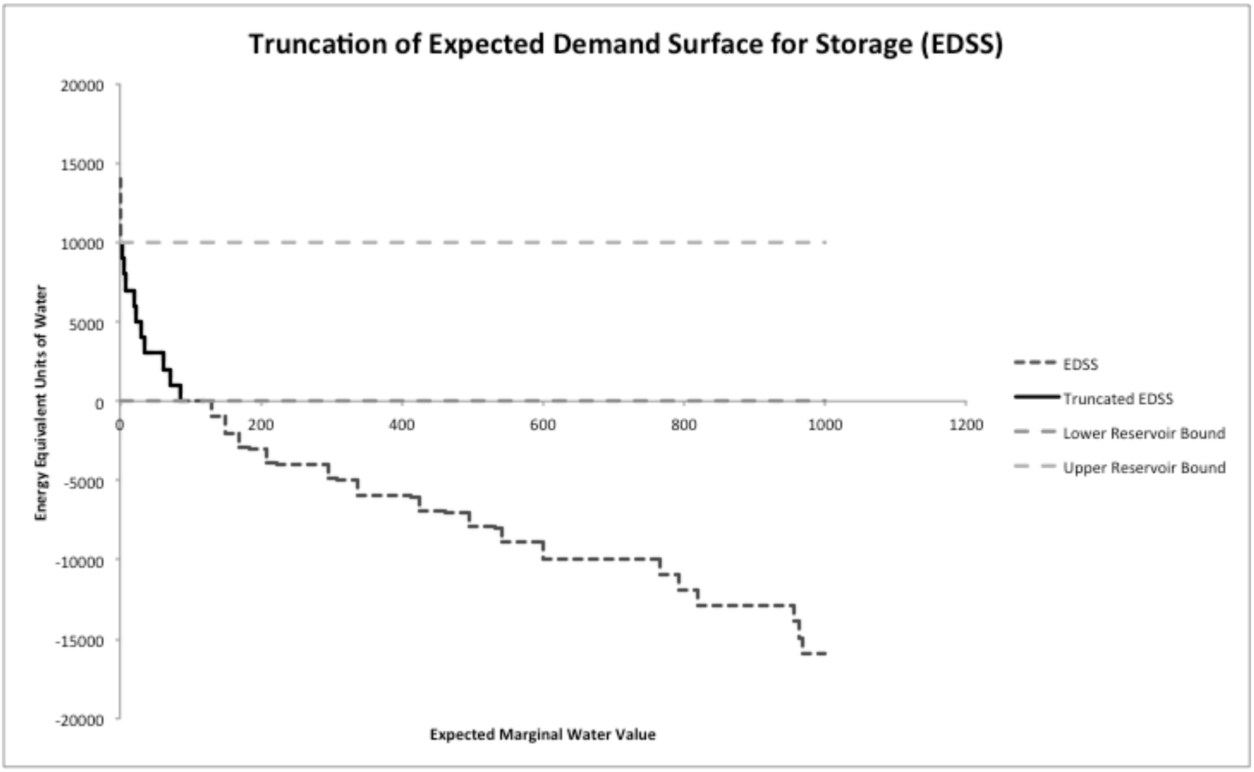


Figure 27: Truncation of EDSS

Given this underlying structure, the cornerwise SCDDP algorithm is based on an examination of how these critical points are altered by the inclusion of stochastic inflows in the problem. In the cornerwise CDDP algorithm the inflow sequence could be represented by a simple shift in the quantity axis. In the cornerwise SCDDP algorithm the inflow sequence must be represented by a shift in the quantity axis combined with a spread along the value axis. The impact of this shift and spread is shown for a single critical point from uncertain inflows on a piecewise linear DSS' below in Figure 28.

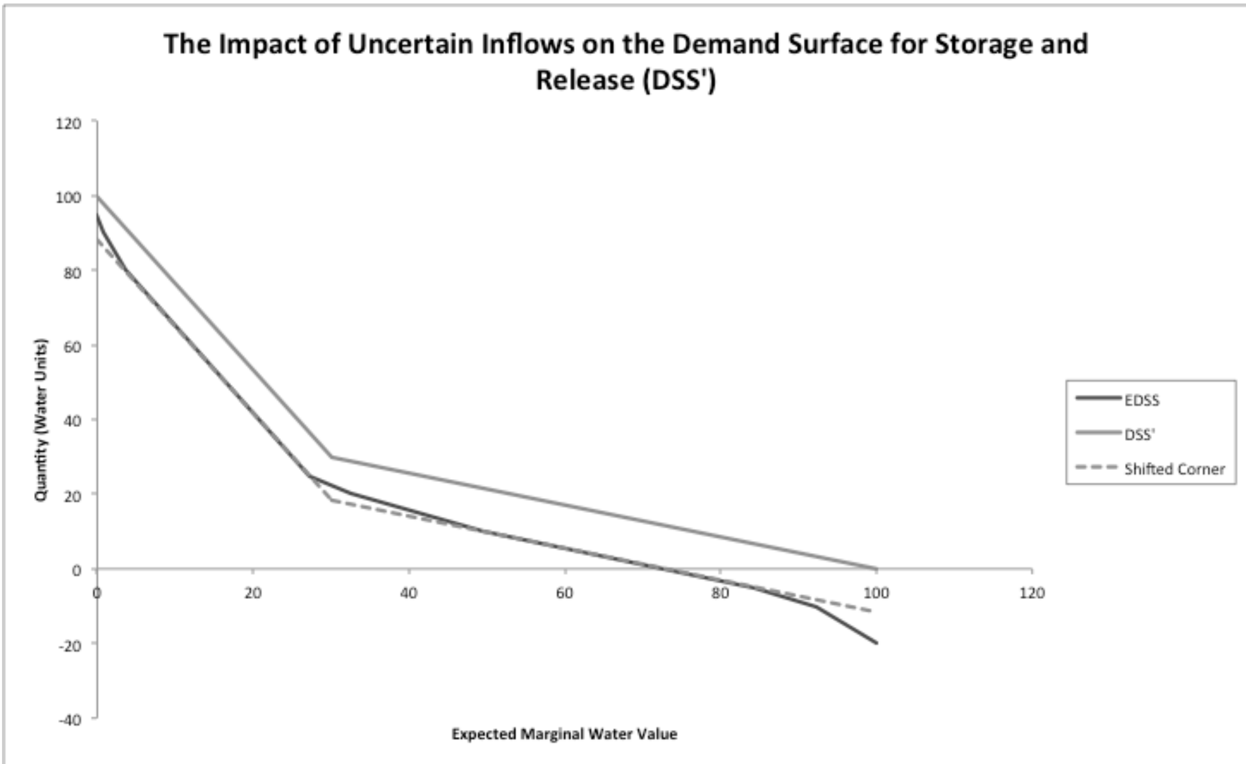


Figure 28: Impact of Uncertain Inflows on a single point in a marginal water value surface

As can be seen above, there are two clear movements that create impact on the critical corner in the DSS'. The most readily apparent is that the curve is shifted by the average inflow. The corner that would be created by only this shift is shown above as the 'Shifted Corner'. However, the EDSS does not have the same characteristics as this shifted corner. Instead the angle that occurs at the corner point is less sharply delineated. Likewise the 'corners' which occur where the EMWV is equal to 0 and 100 respectively have less sharply delineated drop off points. This is what we describe as the 'spread'. Because there is uncertainty about at what MWV the actual corner will occur, there is likewise a spread of each critical corner along the EMWV axis. This combined action implies that rather than attempting to capture the shifted corner point itself, two points need to be recorded to represent each corner point. These are the shifted corner point + ε and the shifted corner point – ε where we assume for some value of ε such that these points represent a small increment either side of the shifted corner point. This method is not entirely accurate as detail of the shifted corner point itself is lost. However, it allows for a continuous piecewise linear approximation of the real MWVS. The slope of the approximate linear piece can be recreated from these two recorded points. An example of this applied to Figure 28 above is shown in Figure 29 below. As can be seen below, for well chosen corner points the difference created by the approximation can be barely distinguishable from the full EDSS.

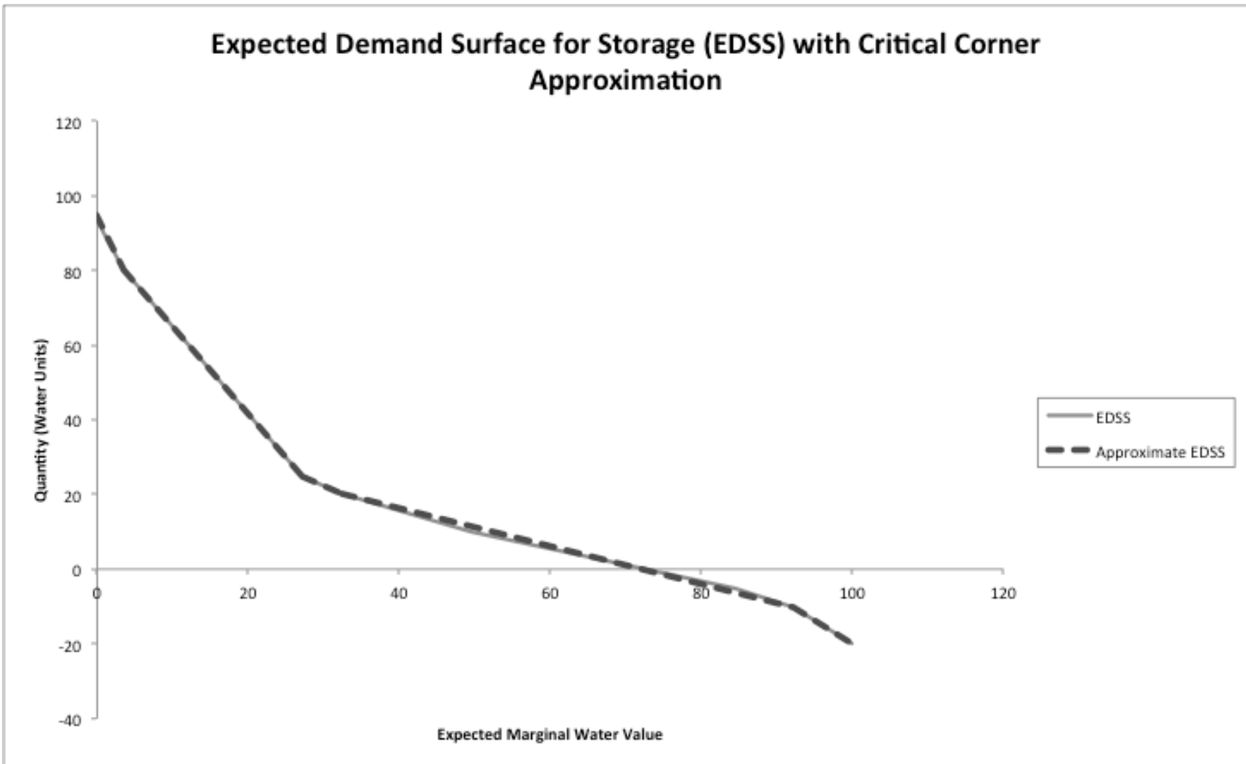
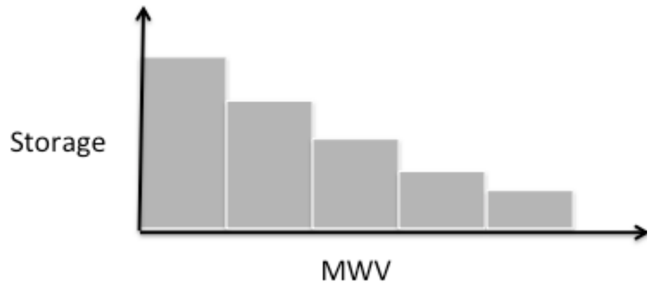


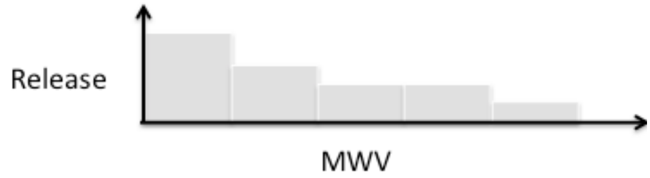
Figure 29: Use of Critical Corner Approximation on Single Corner EMWVS

The formation of the EDSS from a discrete set of stochastic inflows follows the structure described below. This algorithm also includes truncation to represent reservoir limits. A diagrammatic representation is also included below.

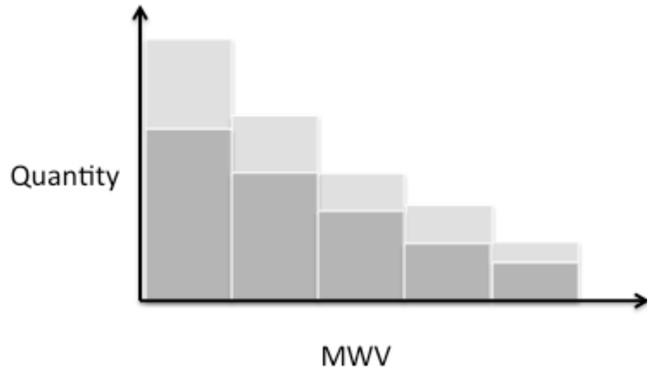
$$DSS_{t+1} = s_{t+1}(v) \quad \forall v \in V$$



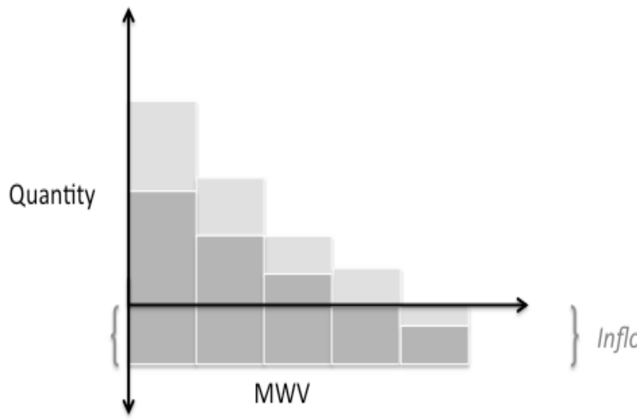
$$DSR_t = r_t(v) \quad \forall v \in V$$



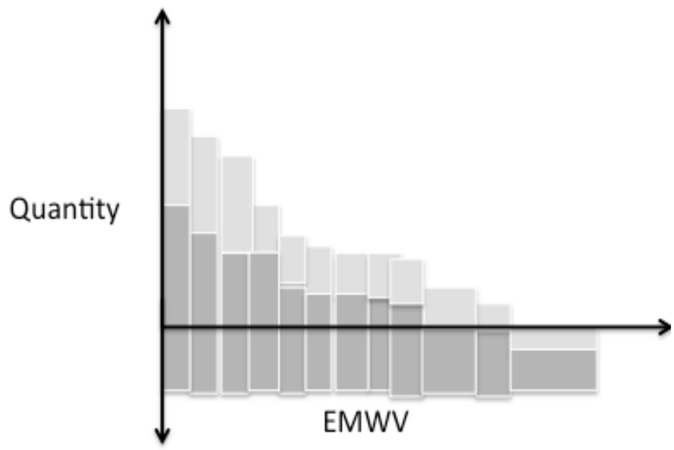
$$DSS'_t = u_t(v) = s_{t+1}(v) + r_t(v) \quad \forall v \in V$$



$$DSS_{t,A} = u_t(v) - F_A \quad \forall A \in Inflow\ Set$$

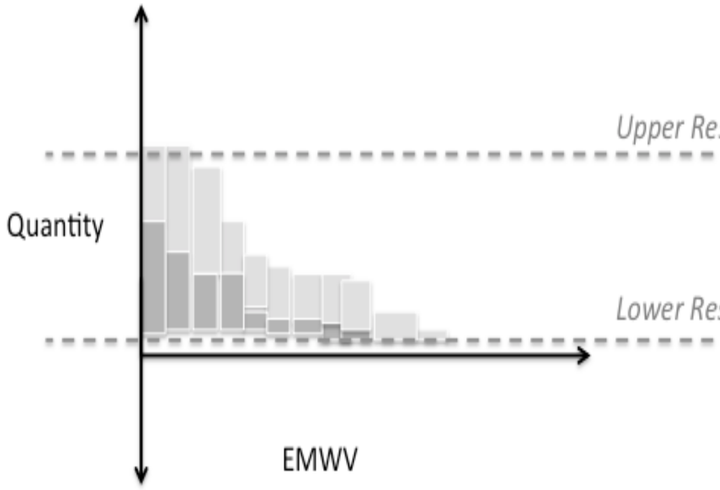


$$EMWV = v(s)$$
$$= \sum_{A \in Inflow\ Set} Probability_A * MWV_s$$
$$\forall\ s\ \in Storage$$



$$EDSS_t = \max(u_t(v) - F, Lower\ Res\ Lim) = \min\ (u_t(v) - F, Upper\ Res\ Lim) \quad \forall v \in V$$

$$where\ EDSS_t\ is\ used\ as\ the\ function\ for\ DSS_{t+1}\ for\ all\ time\ periods\ except\ the\ final\ time\ period$$



These theoretical depictions above have self-evident issues in terms of minimising computational burden as in order to form an EMWV for each storage level to account for uncertainty in the EDSS there is an implicit inversion of the function. This is equivalent to translating the function from the dual space into the primal space before reinverting the function before continuing. In practice our implementation attempts to circumvent some of the issues this implies. This is done by contructing the 'full' expected value curve with EMWV as a function of storage. As we are using a discrete representation of stochasticity this correlated with the diagrams above in that we form a probability weighted MWV for each inflow adjusted storage level.

In our initial implementation at this stage we calculated for every distinct storage level (after inflow adjustment has been applied) an associated EMWV. This was sum of the probability weighted MWV associated with that storage level for each inflow scenario. This addition was relatively straight forward where the storage state was explicitly stated in each of these inflow scenarios. However for many inflow scenarios there were the no critical MWVs in the DSS' for which the associated storage point would explicitly create the required storage level when inflow adjusted. Consequently we estimated the MWV based on linear interpolation on the inflow adjusted storage levels for that inflow scenario. All these probability weighted MWVs associated with the storage level were then summed to form the

EMWV(s). This function would be in turn approximated in terms of critical MWV points as described below and represented in Figure 31.

This approach was found to have a solve time of under two minutes for a two reservoir problem with 5 possible inflow scenarios. However, where we extended the inflow scenario quantity to 30 the additional computational burden resulted in solve times beyond ten minutes. In response to this limitation we revised our approach so that the EMWV is at present only calculated for the critical values which would be produced by a mean inflow scenario, an upper quartile scenario and a lower quartile scenario.

Note that while the representation in Figure 29 is appropriate for forming the EDSS, in order to form the EDSS' for the previous period the DSR must be added to the EDSS. In the deterministic cornerwise algorithm this was facilitated readily as the critical MWV points in the DSR were exactly the same as those in the EDSS. However, the critical MWV points in the SCDDP EDSS have been altered as a result of the stochastic 'spread' and the use of +- ε. The accurate addition of the EDSS to the traditionally formed DSR would require the addition of a further set of DSR based critical points to the EDSS' representation. This would imply, for a cornerwise version of forming the EDSS' in Figure 22 an arrangement comparable to that displayed in Figure 30 below. It can be seen that the approximation performs reasonably well for this EDSS' formation.

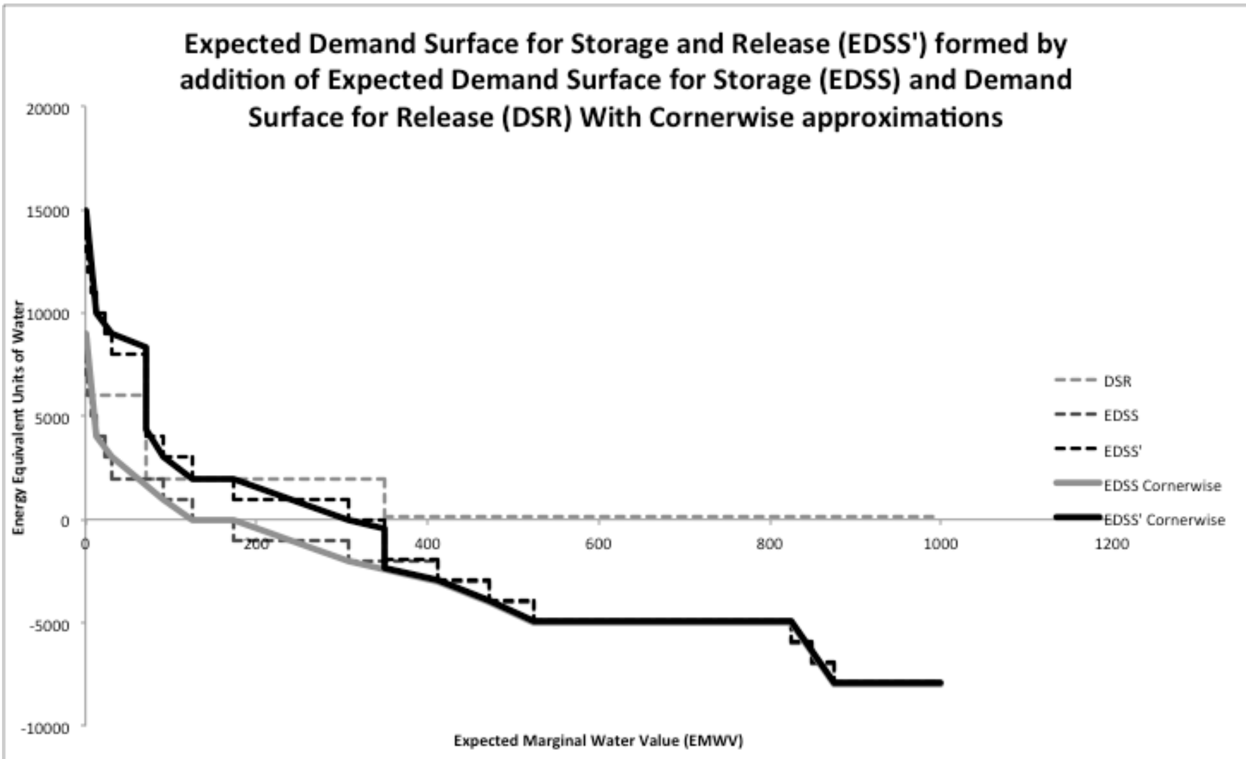


Figure 30: EDSS' formed by addition of EDSS and DSR with Cornerwise approximation.

However, for more complex surfaces the use of both the critical EDSS corner points and those from the DSR would imply that the set of points used to store these surfaces would increase significantly with every iteration. This is because when the EDSS' is passed back to form the inflow adjusted EDSS for the previous period then each of these critical points will result in a critical point + ε and – ε corresponding set of points. This would mean the EDSS for the previous period would include twice the number of critical points as the EDSS', and more than double the number of critical points from the EDSS. This type of increase rapidly becomes unsustainable when the problem size is increased to include multiple reservoirs. The increase is particularly unsustainable due to the implied calculation of appropriate DSR/EDSS values to enable the simple point-wise addition. This interpolation increases the computational burden considerably more than the inclusion of additional points throughout the process.

Three alternatives exist to simplify this to reasonably complexity levels: the EDSS can be reformulated in terms of the DSRs key points, or the DSR can be reformulated in terms of the EDSSs key points, or the entire final set of critical points could be included from the start. The first of these may cause a loss of some of the EDSS detail, however will retain a relatively small set of points to pass back into the previous period, and these points are established during the pre-computation of DSRs. The second of these may cause a loss of some of the DSR detail, will retain a slightly larger set of points to pass back into the previous period, and these points are not established in a pre-computation phase. The third of these implies a pre-computation phase, which would run a computationally intensive CDDP process to develop a set of critical points, and then that set would be used as the basis of the following CDDP calculations. This is only a realistic prospect if the problem is sufficiently small, and there are sufficient different sets of computation to be done using the same base parameters. Each of these possible representations of the EDSS are displayed below in Figure 31.

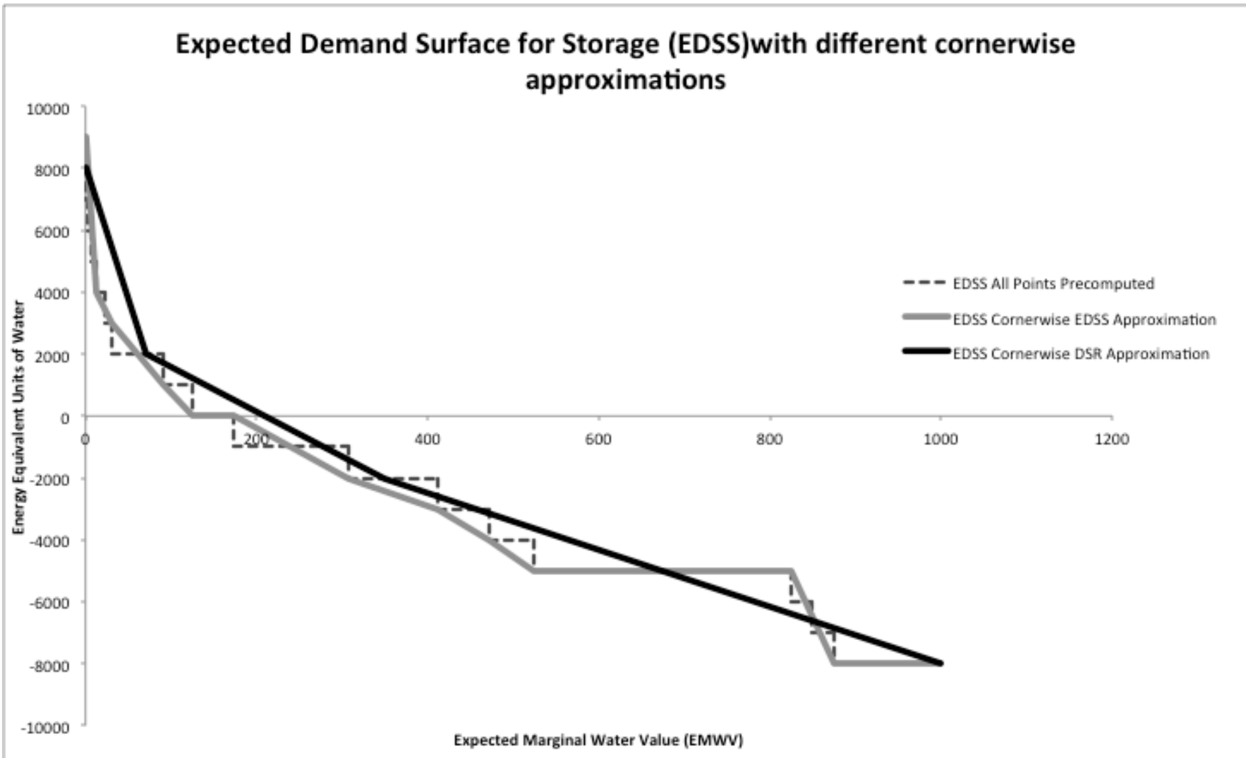


Figure 31: Different Cornerwise Approximations of the EDSS'

It is evident that the EDSS based approximation or pre-computing all possible points is superior in capturing the detail of the EDSS. However, in practice the most influential points for the formation of the start of period DSS' are those which insert piecewise constant sections into the curve. Hence while the EDSS is not best represented through the simplistic use of the DSR critical corners, the EDSS' is fairly well approximated by these points. The points at which it is always known that a large change will occur in the EDSS' are the set of points which represent the DSR, and the points at which the EDSS is truncated. To reflect this we have implemented our SCDDP to reformulate the EDSS in terms of the DSR points and the values just either side of the truncation points. The DSR is then reformulated to likewise include the points for the EDSS' at which truncation occurs. This is recorded by points just a small increment either side of the truncation points. The resultant DSS and DSR can then be added in a cornerwise manner. This is represented graphically below.

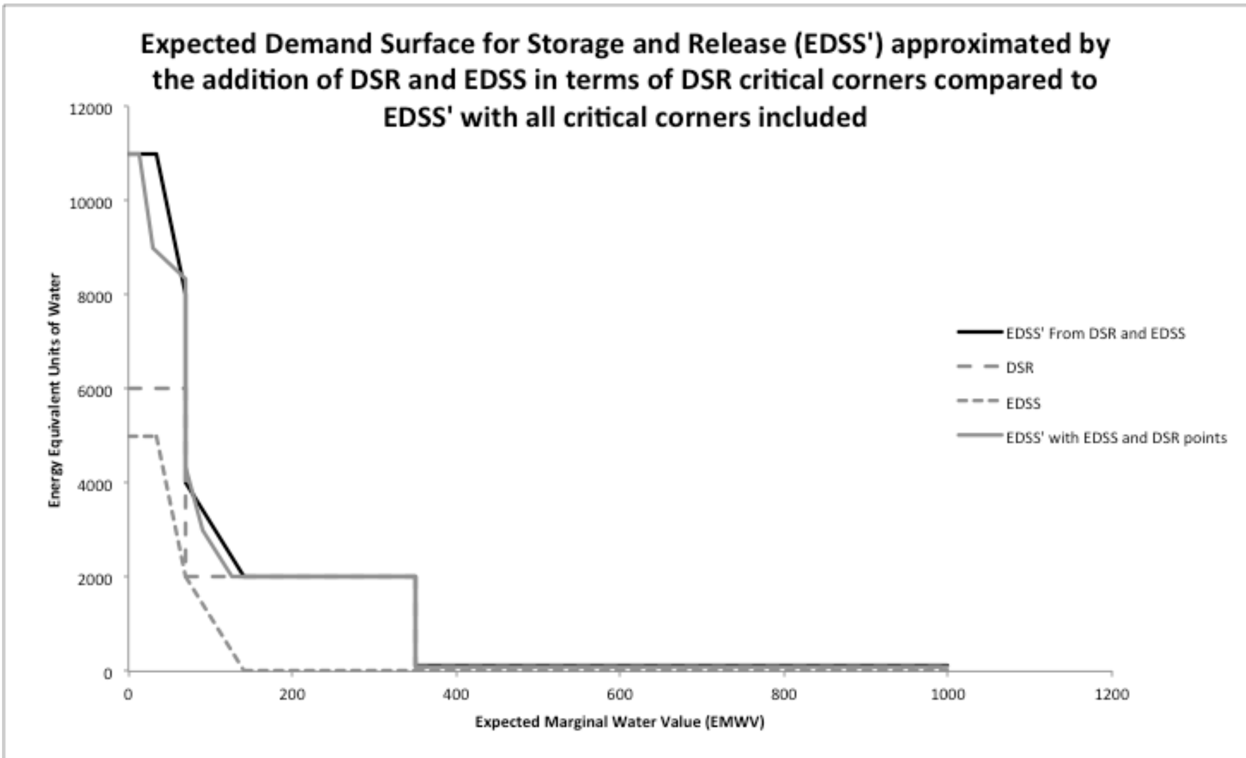


Figure 32: EDSS' approximated by DSR corners and truncation points compared to EDSS' formed from all points

The values for the EDSS' just either side of the truncation point are recorded because in the stochastic case there may otherwise be no indication of the nature of the slope that precedes the truncation point. Hence there would be a loss of information close to the reservoir limits. This is largely a stylistic matter in the single reservoir case, however it has increasing significance for higher reservoir numbers as will be discussed below.

The EDSS' resulting from the cornerwise addition of the EDSS and DSR is the beginning of period demand surface for water for that period. This is the surface which will then be inflow adjusted and truncated to created the EDSS for the previous period.

We note that in theory there exists a wide range of possible truncation methods, however we have adopted an informed policy, in which it is assumed that intra-period actions could mitigate the possibility of violating storage bounds. The informed policy thus only requires that the storage level at each average expected price is within the storage bounds.

In practice this may be an overly liberal policy when compared to real systems that often become risk averse when storage becomes low. However from a theoretical and academic perspective this policy is ideal as it does allow for the possibility of the violation of the reservoir storage constraints and so the reservoir system can be planned such that the risk of violating these reservoir constraints can be managed to an acceptable level. Furthermore the truncation of the EDSS creates a clear definitive truncation point that can be passed back to help form the DSS for the previous period. This definitive truncation point is also essential in order to fully capture the surface of the other reservoir.

One of the motivating factors for the choice of only recording the DSR and truncated points is that in a stochastic system then the impact of a significant change is diffused by the stochasticity. As described above, these inflows both shift and spread the DSS' corner points. This implies that the impact of inserted DSR sections, and the truncation will be spread more generally once the surface is adjusted to take stochastic inflows into account. Correspondingly the loss of information that is the result of reformulating the EDSS section in terms of the DSR MWVs and the new truncation points appears to be of an acceptable level.

A further advantage of employing this simplification is that the truncation points from the previous period are not retained in the new EDSS formation. This means that the number of storage points does not increase with each iteration but remains a constant number of points from the initial DSS formation.

The accuracy of this approximation in performing the inflow adjustment is also impacted by the relationship of the choice of ε to the inflow distribution. Where the inflow distribution has a large variance then ε should also be larger, where the variance is small then smaller ε values will produce a better result.

Other factors, however also influence whether the value chosen is the most appropriate ε value. One of the identified factors which influences to usefulness of the chosen ε value is the angle and proximity of neighbouring corners. These corners are also shifted, and have their own spreading impact on the value axis. Where the spread from one corner overlaps with that of another corner then the EMWV for a given storage multiple corners may influence quantity.

For the purposes of this thesis we have elected to take a simple approach that does not explicitly take into account all DSR, DSS' and correspondingly EDSS characteristic factors in determining the appropriate ε value. In practice we have implemented the cornerwise algorithm with an ε value chosen to be 0.001. However, we note that other factors also influence the usefulness of the chosen ε value and further research could be done to explore the exact nature of each of these. From such research a more comprehensive basis for choosing ε values could be developed. Conversely, it must be noted that the complexity of developing this basis may not warrant the gains in accuracy to be made.

In reformulating the SCDDP algorithm to align with the cornerwise CDDP algorithm outlined above and in R. A. Read, Dye, S. & Read, E.G. (2012)the focus is on choosing the discretisation points more intelligently. The 'curse of dimensionality' will continue to ultimately limit the dimensionality and complexity of these DP based problems. However, through focusing the algorithm on key points this limitation will be reduced and it is likely that the algorithm will be more readily extendible to higher reservoir numbers. A cornerwise algorithm also provides the further benefit that the concepts are intuitively expandable to higher reservoir numbers. This will be discussed in further detail in the section 23 Beyond Two Reservoirs.

10. New Zealand Based Single Reservoir Application Without Inter-Island Links

We have developed a MatLab implementation of the single reservoir SCDDP algorithms. These were initially developed in order to ensure that the algorithm produced credible and replicable results that aligned with previous CDDP and SCDDP implementations. This implementation has been tested using both a set of dummy data with known results for verification, and then with an application based on New Zealand system data. In practice the SCDDP models were extended to explore the implications of incorporating what we describe as a 'double filled LDC'. This 'double filled LDC' allows for a neutral representation of both load and thermal generation capabilities in two islands. This will be described in more detail below before exploring the implications for our New Zealand based single reservoir model.

Our dummy data used a monotone decreasing DCR with 12 thermals filling a single load. In order to verify our model produced the expected results we used an excel model developed using a DCR discretised in terms of reservoir quantities. This model was used as the basis of Dye and Read (2012)To compare the two cases the reservoir size, reservoir truncation, end of time horizon MWVs, DSRs, inflow sequence and number of time periods were identical. In the CDDP case we were able to confirm that both models produced identical results with simply all of these quantities constant. In order to verify the SCDDP implementation it was also essential that the probabilities associated with inflows were equal and that the quantities associated with MWVs and EMWVs were all integer. In doing so we were able to verify that the first EDSS formed through backwards induction is calculated identically between the two models. This confirmed that the DSS is imported correctly and that the stochastic inflows were applied to this DSS as expected. During the addition of the EDSS and the DSR the EDSS is simplified so that fewer points are used for the EDSS' than were in the implementation used for Dye and Read (2012) Starkey etal. However, for every point that is recorded in the EDSS' the calculated EMWV and quantity was produced as expected. This confirmed that the DSR and EDSS were added in the manner expected. All components of the Intra-period and inter-period problems were verified to be working in the expected manner. Deviations occurred in the earlier time periods that were calculated based on this EDSS'. However these deviations are a result of the reduction in critical point numbers used to represent the EDSS in the formation of the EDSS'. This is deemed to be a necessary measure to ensure that the model will continue to be readily generalisable for higher reservoir numbers.

No further benchmarking of accuracy against the Dye and Read (2012)Starkey model was explored. Although there were deviations noted there was insufficient time in the context of this thesis to explore the extent to which our representation of the EDSS' resulted in loss of information. We further note that the specific representation of the EDSS' could in principle be varied in accordance with the nature of system data. Hence our primary aim is to develop a framework in which these representations can be explored, rather than to defend the representation we have chosen.

For our New Zealand based single reservoir model we have assumed that the single reservoir is representative of the combined reservoirs of the Waitaki system. The data that we have used for our New Zealand system is based on a prediction of 2014 system data. This data was provided by associate supervisor John Culy. The initial data set had the available MWs by week with randomized maintenance schedules, and expected demand for each island for each time portion for each week. The time portions are 14 blocks within the week delineated by workday and holiday. The data provided also included expected generation for each time portion for each week. This generation data is based on historical bidding patterns and does not directly reflect SRMCs. Inflows were provided by year, week and scheme for 80 hydro sequences in average MW potential generation.

To formulate the inputs to our SCDDP implementation it was necessary to aggregate expected demand for each island. Using the demand levels within a week we determined for what percentage of the hours in that week the demand was expected to exceed a certain level. These demand levels and associated percentages were used to form a load duration curve. Between these demand levels we assumed the change in percentage occurred linearly.

In our model we assume a simplistic SRMC merit order and so we needed to back-engineer the expected generation data in order to formulate a reasonable estimate of a simplistic SRMC merit order. We first exclude the generation facility or facilities that are the focus of our SCDDP modelling from the generation facility list. The remaining generation sources are separated into groups by island, then within each island by primary fuel type. We assumed that intra-island transfer to fill load was feasible however inter-island transfer was not. For each fuel type we used data from the 2012 Energy data file ("Energy Data File," 2012) and a generic estimated efficiency rate to form a fuel price per MWh. This formed the base quantity of our SRMC for each plant of that fuel type. However, in order to preserve the generation characteristics of our data, further differentiation of generation facilities was necessary. Hence based on the real dispatch as a percentage of available capacity from each generation facility an additional portion was added to the fuel cost. Where a generator dispatched a higher percentage of available capacity this portion was less than where a generator dispatched a lower percentage of available capacity. For each island we formed a merit order list of all non-intermittent generators in that island alongside the percentage of available capacity that was dispatched in the given week. This enabled us to detect any major discrepancies where the plant was dispatched at a much higher rate than other plants with comparable cost, or vice versa. The estimated SRMCs were then adjusted so as to reflect these aberrant behaviours by raising or lowering the SRMC of that generation facility accordingly. Through the application of this method then all thermal generation and all hydro generation was accorded a nominal SRMC. This enabled us to assume that where a hydro-electric reservoir is not being modelled using SCDDP then it can be treated as another form of thermal generation. By calculating a weekly merit order then the general impact of seasonal inflows can be taken into account in modelling these hydro facilities as thermals.

It is assumed that any intermittent generation will always be used to fill a portion of load. The total load filled by each intermittent generation facility in a given week is based on the expected generation of that facility for that week. We make the simplifying assumption that the intermittent generators will on average generate at a constant rate over the week. The portion of load filled by intermittent generation in each island assuming constant generation can then be subtracted from the LDC we have formed for that island. These assumptions greatly simplify the impact of intermittent generation, however, as our purpose is to develop a method for calculating the expected value of hydro release and storage on a weekly aggregated basis then this representation will be sufficient.

Inflow information for each reservoir included both reservoir inflow and tributary inflow. We have assumed that tributary inflow must be used for generation in the period it is received. The majority of hydro generation facilities are treated as thermal generation in our model and so these tributary inflows are taken into account. However for the generation facility we are using SCDDP to explicitly model this is not the case. Hence the load filled by tributary inflow into this generation facility is assumed to be base load and is subtracted from the LDC of the island in which it is located. The generation associated with these inflows is also subtracted from the generation capacity of that facility.

The resultant LDC for each island, and generation facility merit order and generation facility capacities for that island are used as the basis for filling the LDC. Portions of the load for that island are attributed to each generation facility in that island in accordance with the generation capacity of that facility. The facility with the lowest SRMC fills the portion of load that must be filled the highest percentage of the time. This is a simplistic method of filling the LDC that does not take into consideration transmission constraints, ramping constraints, intra-island losses, inter-island transfer or different efficiency levels within each plant. However to prove that the algorithm is functional it is only necessary to provide a simplistic filled LDC. The algorithm is not heavily based on the manner in which the LDC is filled and can readily be adapted to other configurations.

The reservoir or combination of reservoirs that we are modelling in SCDDP, the associated inflows and the generation capacity's are provided by this inflow dataset. However in order to develop a more useful model then the limits on reservoir storage must also be taken into account. These storage capabilities in terms of the generation capacity of each water unit were developed from information on the storage levels, and generation facility efficiencies publicly available on the Electricity Authority of New Zealand's website(OPUS, 2010).

The DSR for each week is formed from the filled LDC for the island in which the reservoir is located. The DSR is formed by assuming that the generation available from our reservoir is equal to the calculated capacity once tributary inflows have been taken into account. This limits the portion of load that is filled as it displaces each generation facility in the merit order of filling the LDC. We also assume that this capacity represents the reservoir release constraints. However, where the reservoir release constraints are more restrictive than the generation capacity of the plant then the release constraints will be used as the limits on the portion of load filled from the reservoir.

At this stage we are considering that each island is isolated and there is no interconnecting links. This assumption will be relaxed after the introduction of the double filled LDC below. However in practice we have run the SCDDP algorithm to model the aggregated Waitaki scheme filling South Island load. This is the base case for our single reservoir model. The reservoir storage limits are based on the values provided in OPUS (2010). For the SCDDP model then we used 30 years of inflow, each with an equal probability of being the inflow. We have assumed that there is no correlation in inflows between weeks. However, the inflows of the different reservoirs on the Waitaki scheme are perfectly correlated so that the information from the same inflow sequence will be used as inflow to each of these reservoirs.

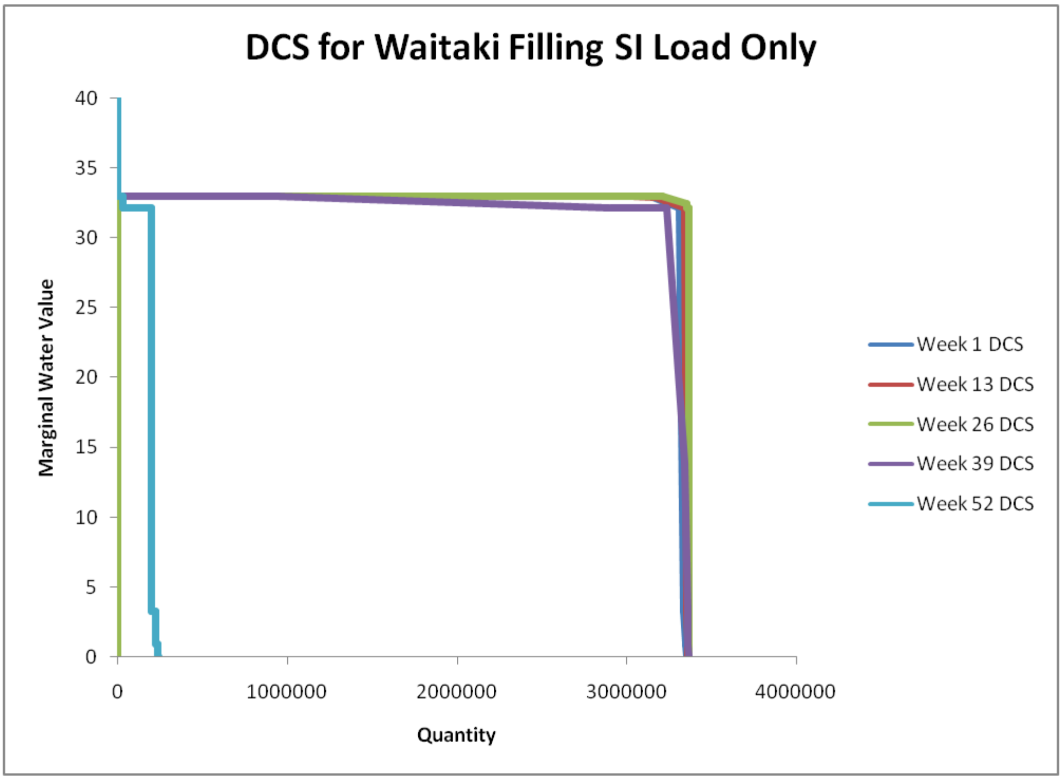


Figure 33 DCS for Waitaki Filling SI Load Only

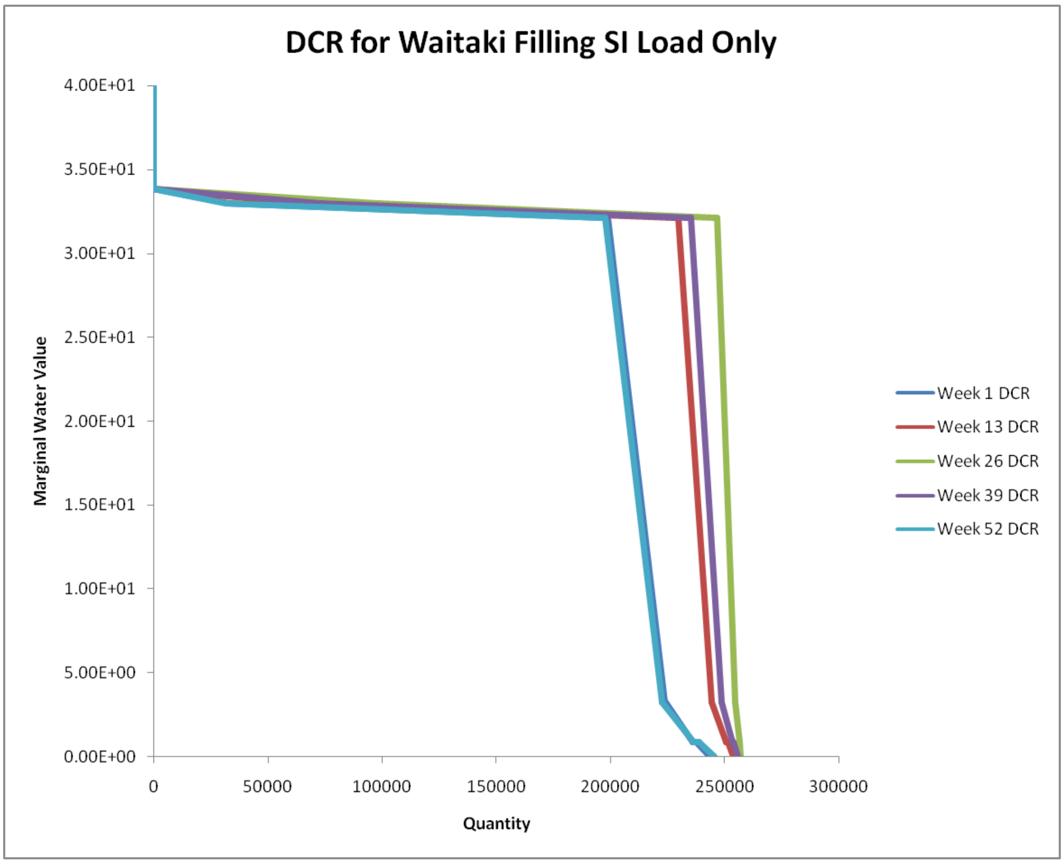


Figure 34 DCR for Waitaki Filling SI Load Only

11. Single Reservoir Load Filling with Inter-Island Links

11.1 Assumptions

In a multi-island system, such as New Zealand there exists a separate LDC for each island. In a multi-island system with inter-island linkages such as the HVDC link then generation can be transferred from one island to the other. Using a double filled LDC is one method that can be used to ensure that generation is transferred from one island to another efficiently. This would imply that over the entire system all load is met in the first instance by the cheapest available generation source. The SPECTRA, two-island implementation of the New Zealand system ignores the complications inherent in this conceptual formulation by making the assumption that there will never be thermals in the South Island (SI), or generation types with similar characteristics. The LDC formulation below is intended to provide a LDC more readily adaptable to scenarios in which there exist alternative fixed cost energy sources in the SI. The double filled LDC is a visualisation of the two LDCs which enables a relatively intuitive interpretation of the inter-island transfer and the impact of these inter-island links on the reservoir release and storage surfaces.

For one of the single reservoir New Zealand representations below, we assume that there exists one reservoir in a single region that is contributing to the filling of two LDCs, one local LDC, and the other in another region. However, to understand the underlying concepts we will slowly build up to this degree of complexity. In the first instance we consider a case in which all generation sources in both islands are fixed and transfer is enabled by a quantity constrained link with no losses. We will then consider the impact that the inclusion of losses with have on this double filled LDC. Then, once these representations have been established we will consider how the inclusion of hydro-electric generation impacts on a double filled LDC. This leads naturally to a discussion of how the double filled LDC in turn alters the computation of the DSR for that hydro reservoir.

A double filled LDC comprises of two localised LDCs with transfer between. For the purposes of discussion we will describe one of these LDCs as being the local region, and the other as being in the distant region. Where there are no hydro-reservoirs in either region then in practice the impacts of the HVDC link in each area are symmetric and so we will only discuss them for the local region. During our discussion of the inclusion of a hydro-electric reservoir we will assume that the reservoir is positioned in the local area and discuss the impacts in each region separately.

When the DSR is formed for a local reservoir in a system with a double filled LDC the quantity of load in the distant region that can be met by generation from the local region is constrained by the flow limits on the inter-regional link. Therefore there will generally be some load which is in the distant region that can never be met by transferred load and this load will not be explicitly included in reservoir release decisions from reservoirs in the local region.

The losses and transfer constraints that exist in the system are internalized in the formation of the DSR from the double filled LDC. This means that once the DSR is constructed the process is identical to the initial single reservoir algorithm outlined in above, and could be applied to any DSR, even those formed from non-monotone LDCs and LDCs with non-coincident peaks. The construction of the DSR is the only substantial difference between a single reservoir supplying multiple LDCs, and a single reservoir supplying a single LDC.

11.2 Double Filling the LDC

11.2.1 Double LDC with no Transfer and Losses, or Hydro

The visualisation of multiple, separate LDCs, and the establishment of a merit order for filling them is a matter fundamental to the ability of a model to take into account load requirements in different areas. The advantage of using a double filled LDC is that the impact of transfer on the merit order can be viewed in a symmetric manner. In effect appending the two LDCs from each region creates the double filled LDC. The initial representation below does not allow for any transfer between the areas. Figure 35 indicates the distinct LDC for each island. **Error! Reference source not found.** represents the two LDCs together in a single vertical diagram that would allow for intuitive illustration of links between the load areas. In the below diagrams the area in green represents the local LDC, the area in red represents the distant LDC. The subsections comprising each LDC represent fixed-cost generation facilities that are filling these respective LDCs.

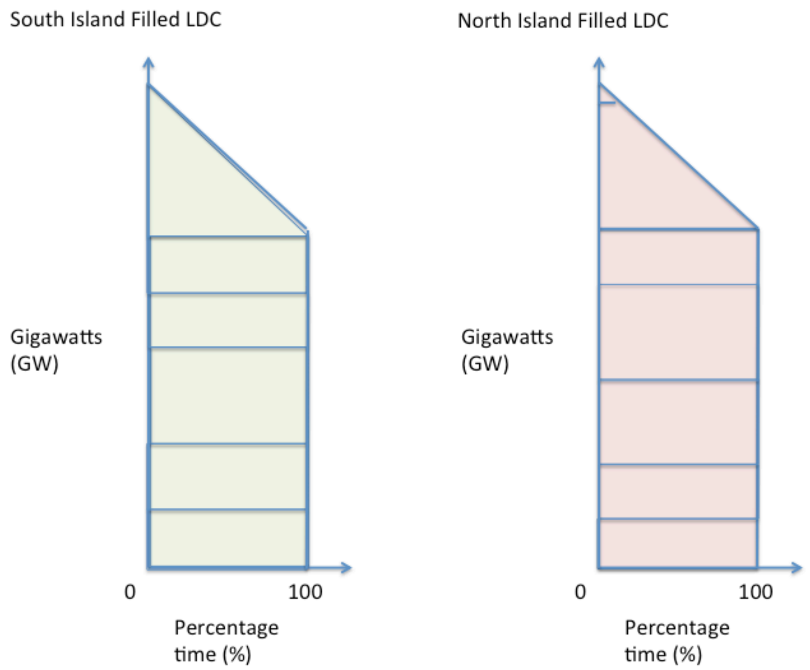


Figure 35: An LDC for the South Island (local) and North Island (distant)

By rotating the LDCs and allowing an equivalence point between the LDC peaks to become the vertical axis we can form a neutral representation of the load requirements represented in Figure 36. The filling of each load by generation facilities would then take the form of diagonal subsections of these load areas. Note that for the purposes of this initial explanation we will not further clutter this discussion by the addition of nominal SRMC for generation. The more general representation implied by the horizontal form would not be altered to reflect which LDC we consider 'local' and so is more appropriate for extension to the case where there are reservoirs in both islands which will be addressed later in this thesis. For the purposes of this report, the representation included in Figure 36 below will be referred to as the horizontal double filled LDC.

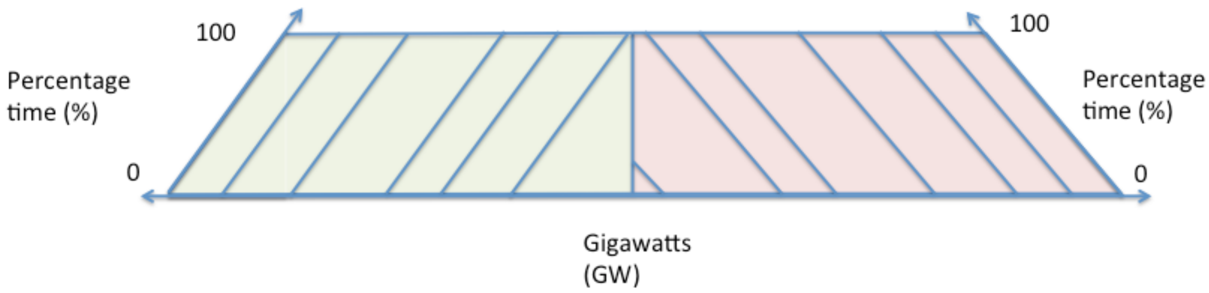


Figure 36: Horizontal double filled LDC with no transfer

Where transfer is possible, then energy can be transferred to fill load from the other area. In the present case we assume there exist no hydro or losses in the system, and only a one-directional transfer link allowing local power to fill distant load. This allows excess generation capability not used for filling the local LDC to then be transferred to fill distant load. To demonstrate this method we have assumed that the peak load requirements between the two islands are coincident. This implies that when these distant generators are not generating for distant load then they are available to fill local load. Effectively by allowing transfer, this enables facilities with universally cheaper generation to utilize full generation capacity to be utilized throughout the time period. This is both the case for the purposes of demonstration and is a base assumption in our implementation. A horizontal double filled LDC with one-directional links and coincident peaks is shown in Figure 37.

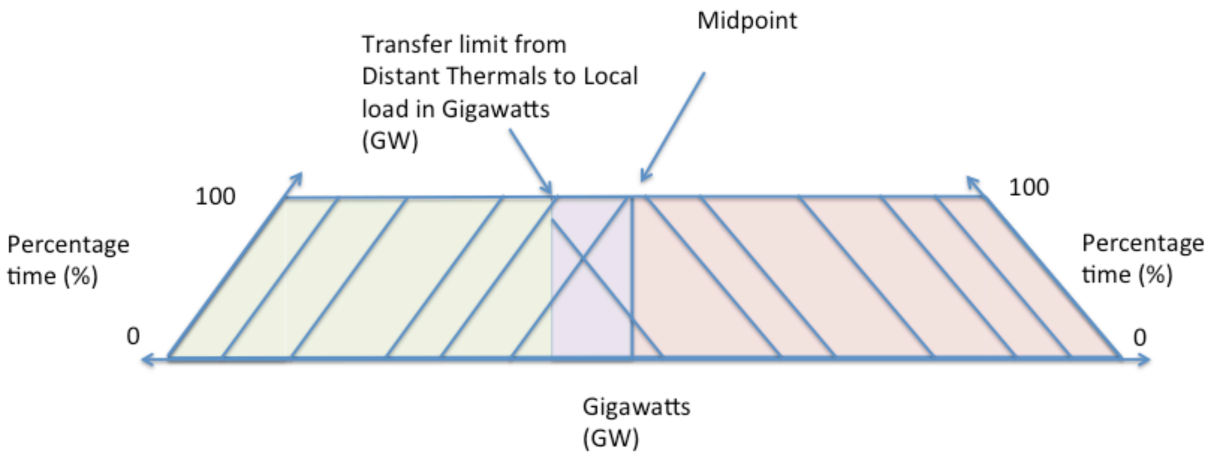


Figure 37: Horizontal doubled filled LDC with one directional transfer

In the above diagram, the lines delineating the subsections of load filled by thermals in the distant area can be seen to continue into the local LDC area. These lines are then constrained by the transfer limit as indicated above. It is where this extension of thermals into the local area occurs that load can be filled either by the local or the distant thermal. In this case it is whichever of these thermals can fill the load at the lowest marginal cost that will generate in practice. The comparable representation with two way transfer on the horizontal double LDC is shown in Figure 38 below. In this case, not only do the thermals from the distant area extend into the local area but also vice versa. Thus in this entire area of overlap, the thermal with the lowest SRMC will generate to fill the load.

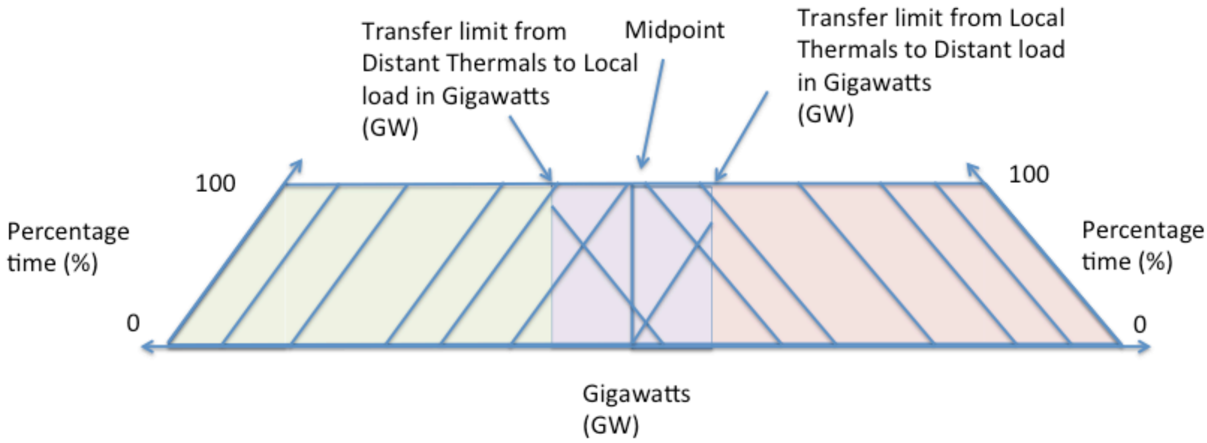


Figure 38: Horizontal double filled LDC with two-directional transfer

In filling the horizontal double LDC with transfer then the thermal with the lowest SRMC available in a given segment is used to fill that segment. An example of this with nominal marginal costs is shown in Figure 39. For example in the section where the load can be supplied from a distant area at a cost of 6 or a local area at a cost of 12, and there are no losses, then the energy will be generated in the distant area with the cost of 6. The chosen generation source to fill each section of load is represented by the darker sections in the diagram below. Note that where the SRMC of generation in both islands has a nominal value of 10 for filling the same portion of load that we are indifferent as to which generation is used to fill that load. The portion of load for which we are indifferent is represented by a darker purple section in Figure 40.

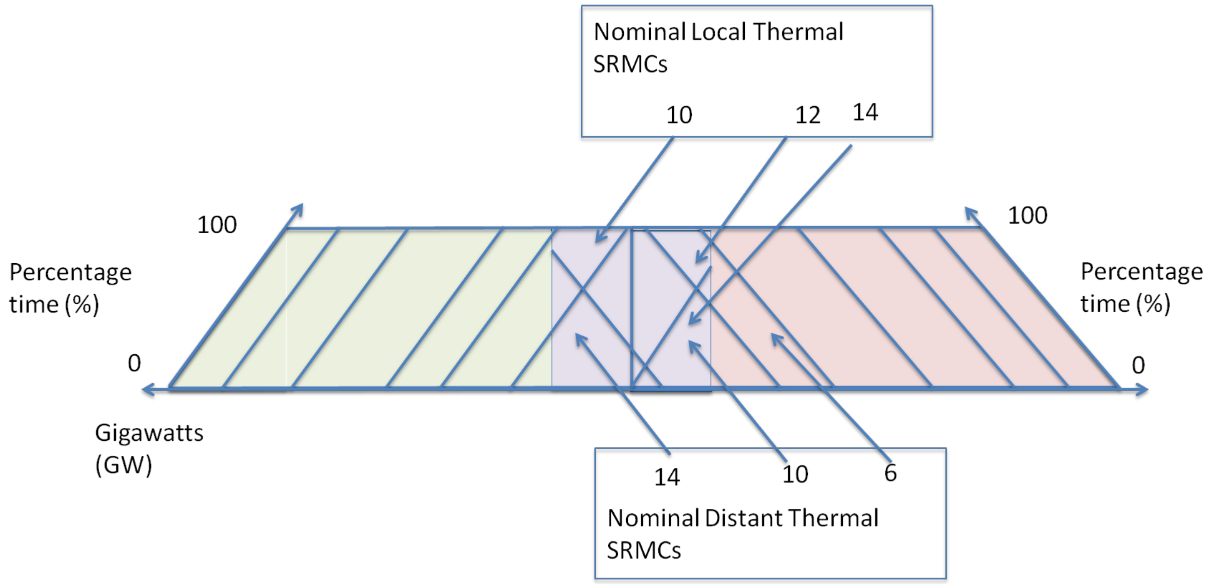


Figure 39: Horizontal double filled LDC with transfer and nominal SRMCs

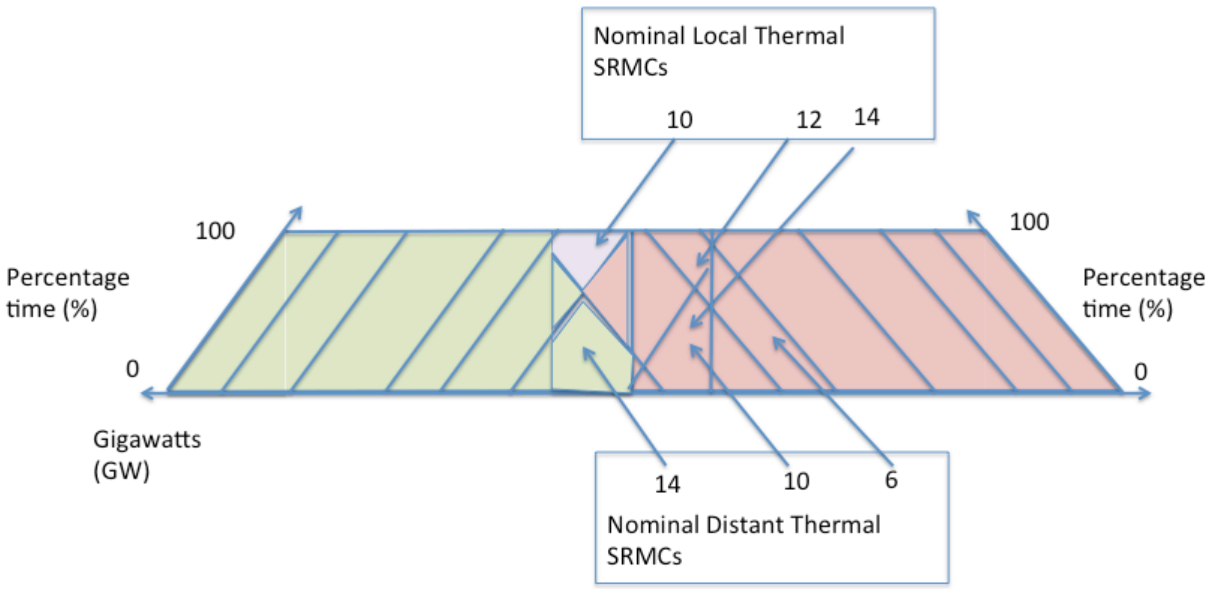


Figure 40: Horizontal double filled LDC, indicating actual transfer

A double filled LDC can also be used to represent cases in which load peaks are coincident, or in which they are partially coincident. However exploration of these representations is beyond the scope of this thesis.

Simply because a form of generation is too expensive to be considered to fill local load, it does not exclude it from being a useful resource for filling load in the distant region. This generally occurs when there is a large quantity of cheap generation resources in one island, and a scarcity of cheap generation to fill load in the other. In theory similar diagrams can be developed for partially coincident peaks and any variation in between. However, for the sake of simplicity the remainder of the discussion will continue to focus on the case with non-coincident peaks described above.

11.2.2 Double LDC with Transfer and Losses, No Hydro

The double filled LDC diagram can also be adapted to account for losses inherent in real transfer links between islands. For example, where 10 GWh of generation have a particular SRMC in the local island then transferring that 10 GWh of generation through an interisland link with a 10% loss factor will result in only 9 GWh arriving at that price in the distant island. This means that the price that is paid in the distant island to receive 10 GWh from that generation source is calculated using the algorithm below:

$$Given\ a\ Q_{generated}, SRMC_{generation}\ and\ Loss\ rate$$

$$Q_{received} = Q_{generated} \times (1 - Loss\ rate)$$

$$Cost_{generation} = Q_{generated} + SRMC_{generation}$$

$$Cost_{generation} = Cost_{supply}$$

$$SRMC_{received} = \frac{Cost_{supply}}{Q_{received}}$$

The impact of losses is that the quantity the thermal is able to supply in total to the distant island is less. However the price that is received by the generator per generated GWh must be the same. Consequently the price paid by the consumer per GWh that is received must be higher. A facility that generates 30 GWh in the local island with a nominal SRMC of 10 per GWh to supply load in the distant island over a link with 10% losses would only supply 27 GWhs to the distant island. The price for each GWh received in the distant island would then have to be 11.11 per GWh to ensure that the SRMC was recovered by the generator. The impact of losses on the double filled LDC diagram is that the area supplied by a thermal changes shape as it crosses the equivalency point. The change in shape corresponds to the reduction in the total quantity of power that can be supplied. It can be seen in the diagram below as losses cause the thermal generator capacity to be delineated by lines at a more acute angle after crossing the midpoint. This corresponds to a reduction on the Gigawatts axis.

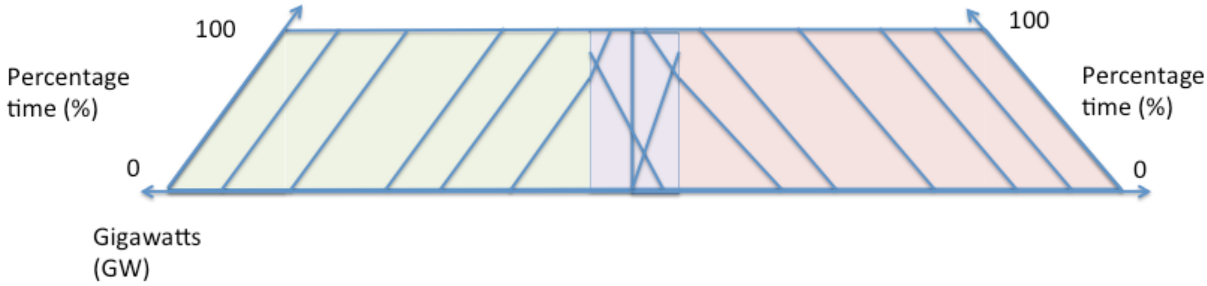


Figure 41: Horizontal double filled LDC with transfer and losses

Equivalently the losses could be considered as an alteration in the efficiency with which the power is supplied to the grid from that thermal generator.

In either case the impact of losses will correspond to a higher effective cost per GWh of the load that is filled by the loss-adjusted segment. Fewer units are available for supply, but the same number are generated. This implies an increase in the cost of each unit which supplies load through a link with losses. Where there is a transfer available, the merit order for filling that section of load must always be based on the cost of filling that load with a particular source of thermal generation in local terms. Where the SRMC of two thermal generation facilities are equal within their respective local segments, however one was in the distant island and incurred losses through transfer to fill a given segment of load, then the local generation would be preferred. This means that where there are transfer costs the merit order is based on is the cost per GWh once they have been taken into account. The darker sections in Figure 43 show the implication of these losses for the preferred choice of generation in the filling of a horizontal double LDC. Notably on the diagrams below the level of transfer has been reduced. This reflects that the line constraints constrain the quantity of electricity that is supplied to the line. This is the same as the quantity that is being generated for transfer, but is a larger quantity than the quantity that will arrive to the other island. Consequently as the LDC below is in terms of demand supplied the transfer capability is implicitly reduced.

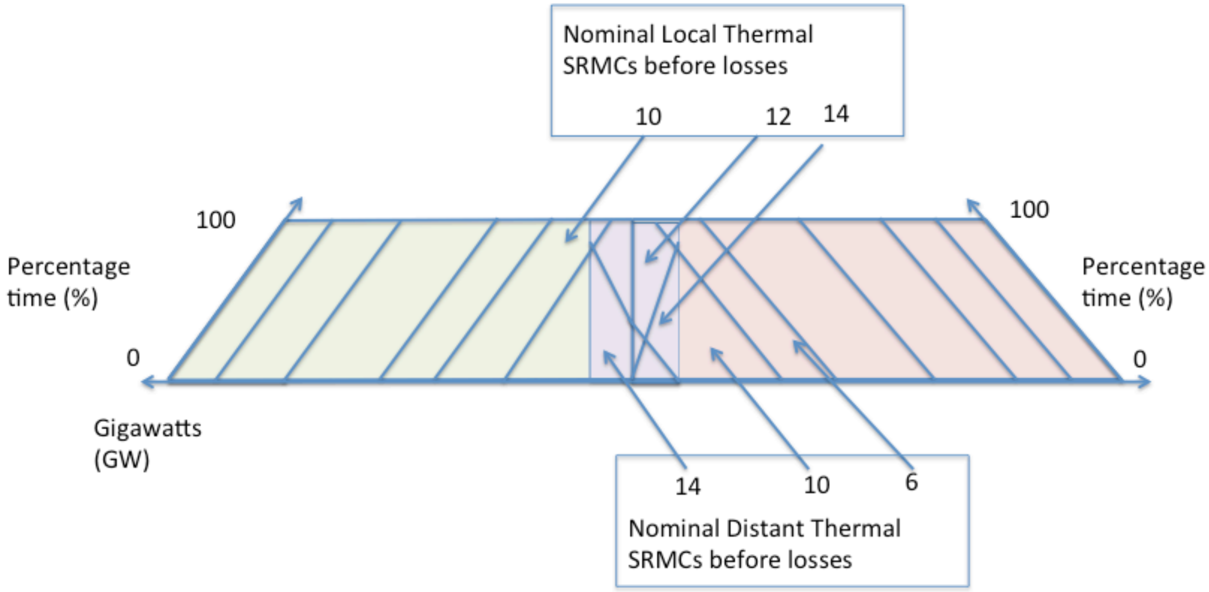


Figure 42: Horizontal double filled LDC with transfer, losses and nominal SRMCs

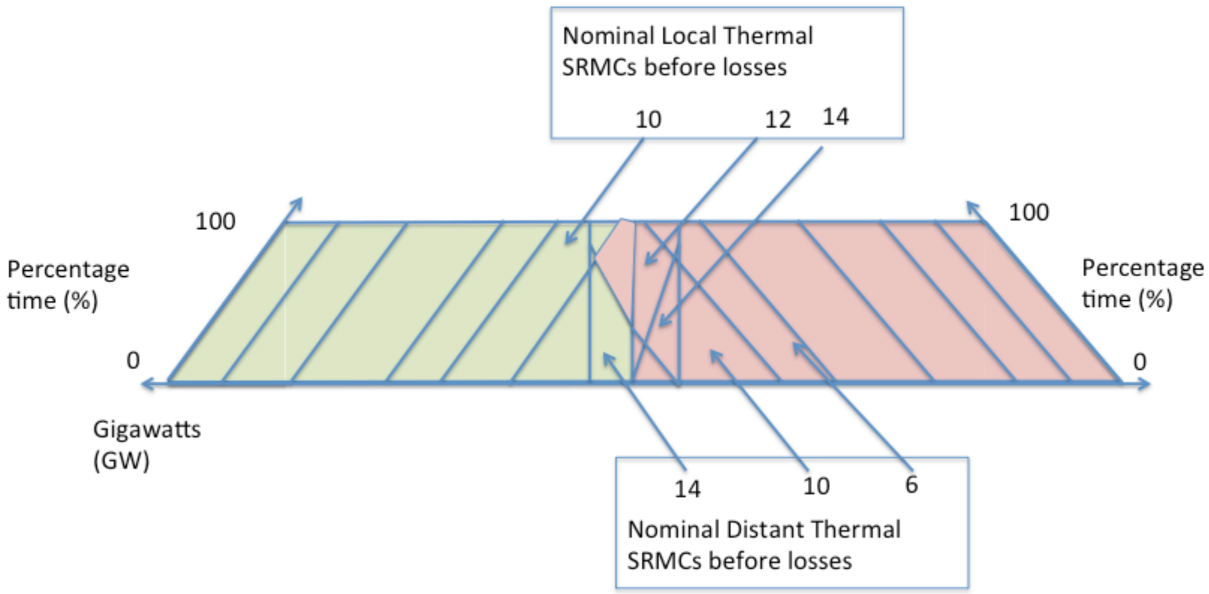


Figure 43: Horizontal double filled LDC with losses. Indicates actual transfer.

As can be seen above, where two sources of generation have equal SRMCs within their respective localities then the local one will be chosen when losses exist in the system. Whereas where no losses occur we are indifferent between utilizing the two sources. This can be seen above where there is a local thermal with a nominal SRMC of 10 before losses and a distant thermal with the same nominal SRMC before losses. However, if there were a nominal SRMC in the distant island of 9 before losses are applied, the application of losses would mean the effective SRMC in filling local load is the nominal value of 10. This means that where there is a local thermal with a nominal SRMC of 10 before losses and a distant thermal with a nominal SRMC of 9 before losses then we are indifferent as to which of these generation sources we use to fill that portion of load.

11.2.3 Double LDC, with Links, Losses and Hydro

The incorporation of a hydro-reservoir into the filling of the LDC is in many ways trivial from the perspective of the LDC itself. A hydro reservoir is, like any other generation source, located in a single area and can supply a component of the load. It is treated like any other generation source that fills the LDC based on the merit order of the associated SRMCs or MWVs. The area with the hydro capacity will be considered as the local island for this discussion.

Displacement that occurs in the filling of the local component of the LDC is largely identical to where a single LDC is available as is discussed above. However, this displacement alters the extent to which more expensive generation sources are used to fill local load. Correspondingly where transfer is available this displacement also has implications for the filling of the distant LDC. The inclusion of a hydro facility in filling the local LDC has freed up some cheaper generation capacity in the local island that can be to replace load transferred from the distant island, or could be transferred and used to fill load in the distant island. This may mean that the less distant generation is used to fill local load, that more local island generation is used to fill load in the distant island, or simply that the generation which is transferred has a lower cost. In any of these cases then the total cost of filling the load in the system will be reduced. In the first case this will decrease the utilization of the link between the two areas. Likewise in the second case this will also increase the utilization of the link between the two areas. The effect of displacement can be seen on the LDC depiction below in Figure 45, where the system moves from distant resources filling load in the local island to a system state where the load in each island is being sourced from that island.

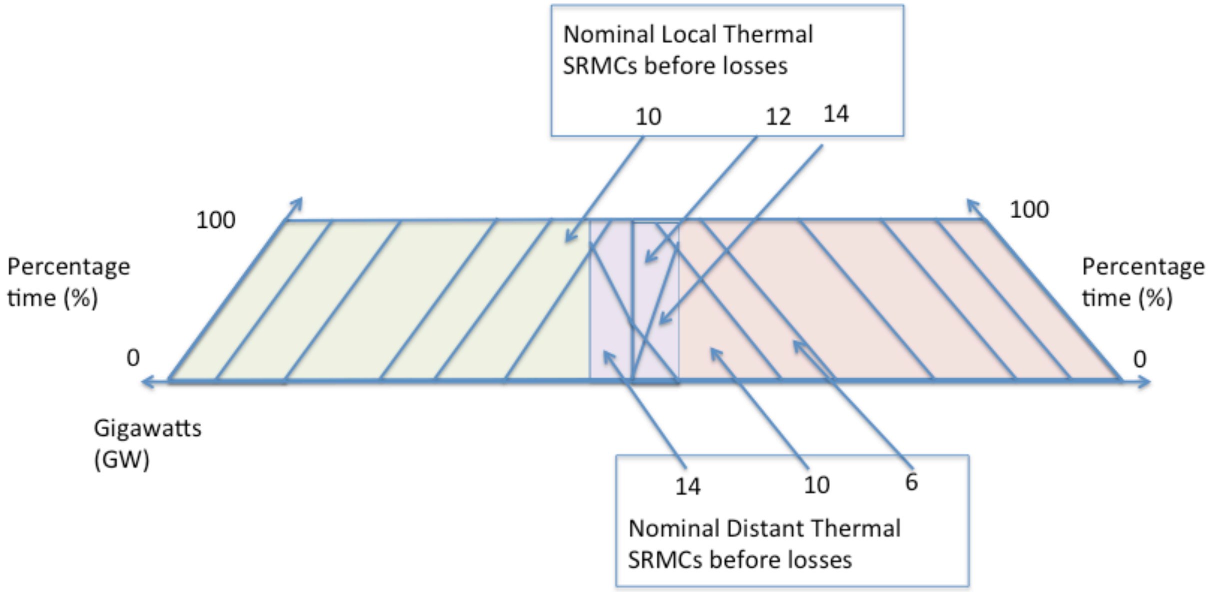


Figure 44: Horizontal double filled LDC with transfer and losses and no hydro

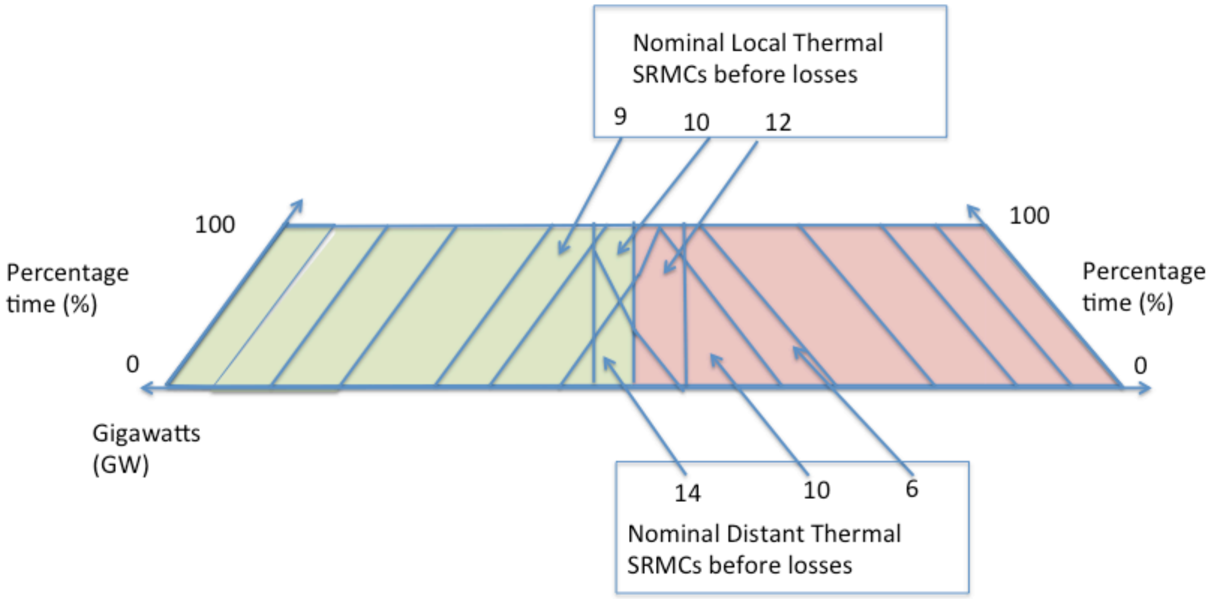


Figure 45: Horizontal double filled LDC with losses and hydro, showing actual transfer

Where the SRMC of generation sources in the island with the hydro reservoir in the local island remain higher than those in the distant island even after displacement, the only impact on the system is a reduction in total cost. The additional complexity from including hydro in the system is very low, however the impact of transfer on the storage and release surfaces for the hydro-reservoir need to be explored in light of this framework. The implications of this will be further discussed through forming the DSRs for a hydro-reservoir faced by a two LDCs linked by a capacity constrained line using this double filled LDC visualisation method.

11.3 Forming DSRs

Once a double filled LDC is created, the transformation of data into a DSR is relatively straightforward. A DSR is created to reflect all possible critical thermals costs faced by the reservoir. There are three possible sets of cost levels, depending on the position of the reservoir in the merit order. The instance where the hydro-electric generation displaces a thermal which is solely used for generation in the local island will result in the a critical thermal SRMC equivalent to when only a single LDC were being filled.

The instance where the reservoir displaces a thermal that is partially filling local load requirements and partially filling load requirements in the distant islands implies two different cost levels at which the thermal can be displaced. Displacement can occur with respect to the local component at the local critical thermal SRMC, whereas the displacement of the component in the distant island will be displaced at the loss adjusted critical thermal SRMC. The quantity that is generated in order to fill this load will be higher than quantity of the load that is filled.

Finally there is the case where the reservoir displaces generation that has been transferred from the distant island. Here again two critical prices are necessary. The first of these is the critical loss-adjusted price at which the generation is available in the local island. This is where the hydro fills local load rather than the load being filled by imported generation. The second price is the critical price at which it becomes worthwhile to transfer the hydro generation from the local island to the distant island in order to displace the distant thermal in the distant island also.

In summary, the critical thermal costs faced are:

- the local values for local thermals
- the loss adjusted values of exporting energy to the distant area
- and the loss-adjusted values of importing energy from the distant area.

The distant areas thermals are backed off from supplying local load at the loss adjusted cost of importing energy; this is the MWV at which local hydro may displace distant thermals supplying local load. The distant thermals can also be backed off from supplying distant load at the loss adjusted cost of exporting energy; this is the MWV at which the local hydro displaces the distant thermals in supplying distant load. The local values are the MWVs at which local hydro displaces local thermals in the merit order for filling the LDC.

The associated release quantities with the local values for local thermals and loss-adjusted values of importing area from the distant area are based entirely on the displacement of that thermal in the merit order. However, the total quantity of generation filling load in the other island must be constrained to reflect the limits of the HVDC link. And consequently the quantity of release associated with that merit order displacement can be constrained.

12. New Zealand Based Single Reservoir Application With Inter-Island Links

In constructing the New Zealand application in MatLab using the double filled LDC technique described above we based our implementation strongly on the single LDC application described above. The only clear delineation was that a capacity constrained link was used between the two islands with a capacity of 1000 MW and a transfer loss factor of 0.9 on transfer from one region to another. This is internalized into the LDC fill as described above. In brief, for filling the more expensive components of each island's load then the cheapest out of either loss-adjusted generation transferred to the island or generation within the island is used to fill the load. There are two different cases considered for the island in which the reservoir is located. The first is which generators fill the most expensive local and distant island load where the reservoir is the generator with the lowest SRMC. The second is which generators fill the most expensive local and distant island load where the reservoir is the generator with the highest SRMC. Based on this information a merit order for filling the load can be created from the double filled LDC.

The solutions which were developed for the Waitaki scheme, modelled with the same characteristics as in the single reservoir cases above except with the inclusion of the double filled LDC are as indicated below. We note that there was no external verification of the LDC fill and DSR development component of our MatLab implementation. There is no literature which currently describes or implements a double filled LDC. Consequently we can only be assured that the results of our double filled LDC implementation comply with our expectations.

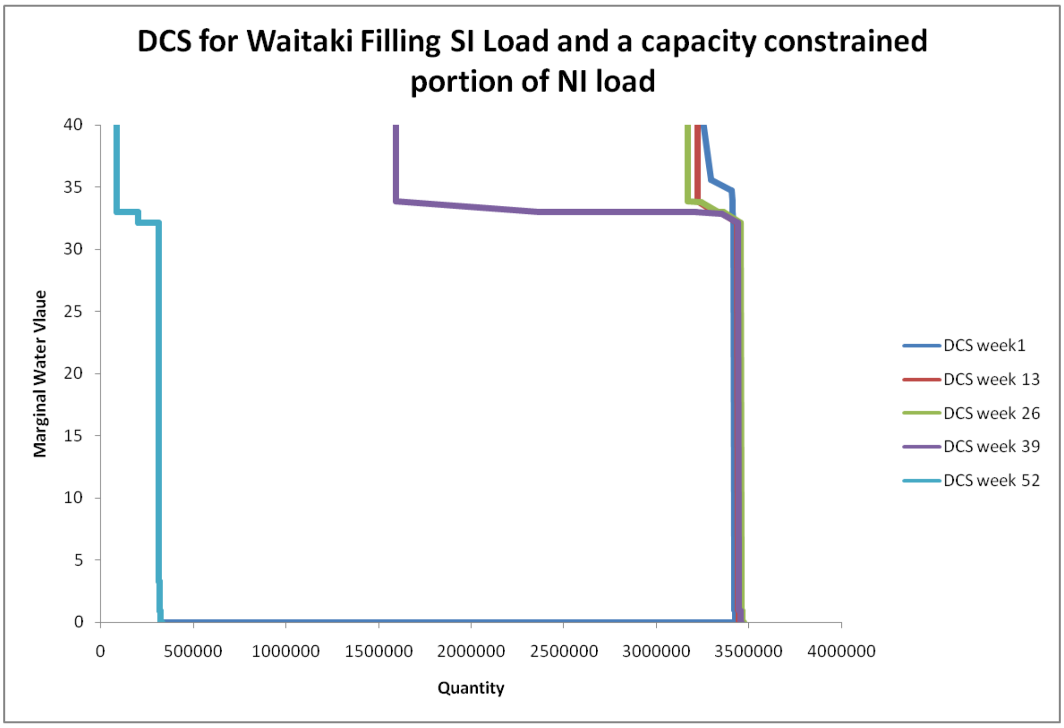


Figure 46: DCS for Waitaki Filling SI Load and a capacity constrained portion of NI load. Same scale as Single LDC fill in earlier corresponding Graph.

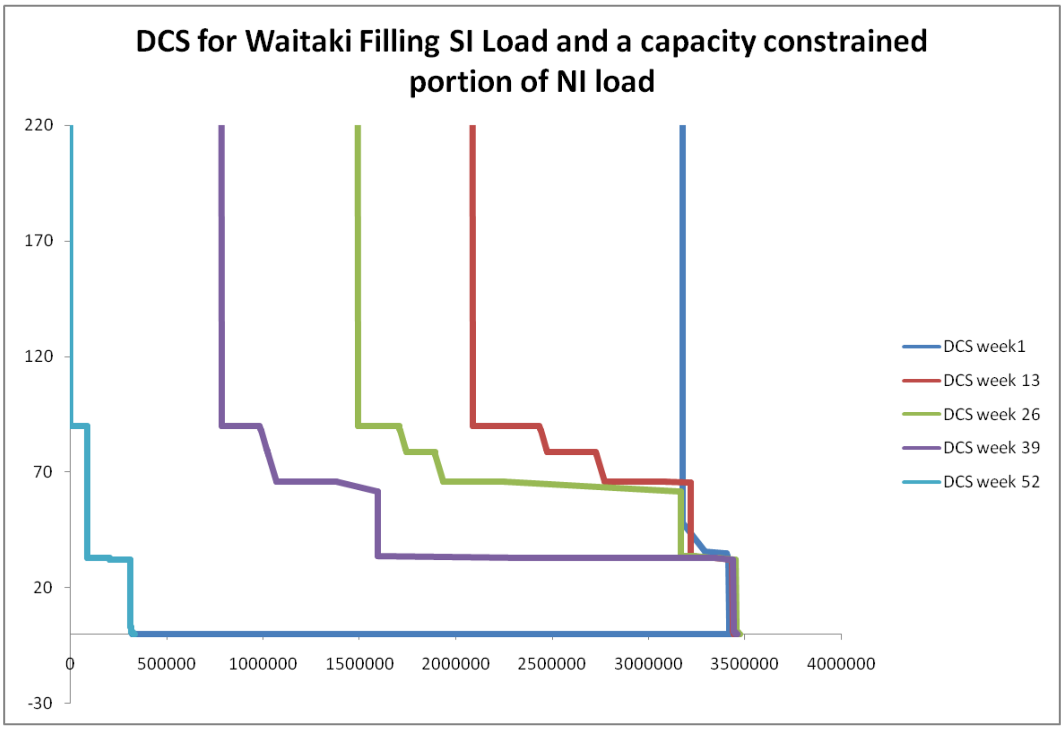


Figure 47: DCS for Waitaki Filling SI Load and a capacity constrained portion of NI load. Shows detail of Figure 34.

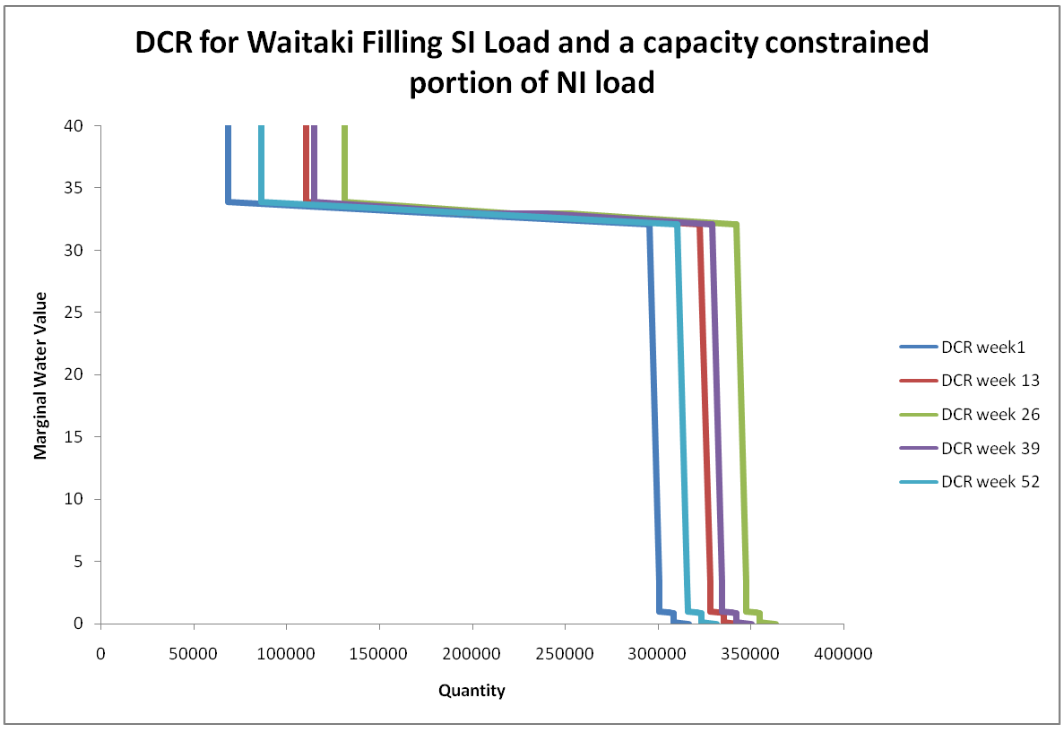


Figure 48 DCR for Waitaki Filling SI Load and a capacity constrained portion of NI load. Same scale as Single LDC fill in earlier corresponding Graph.

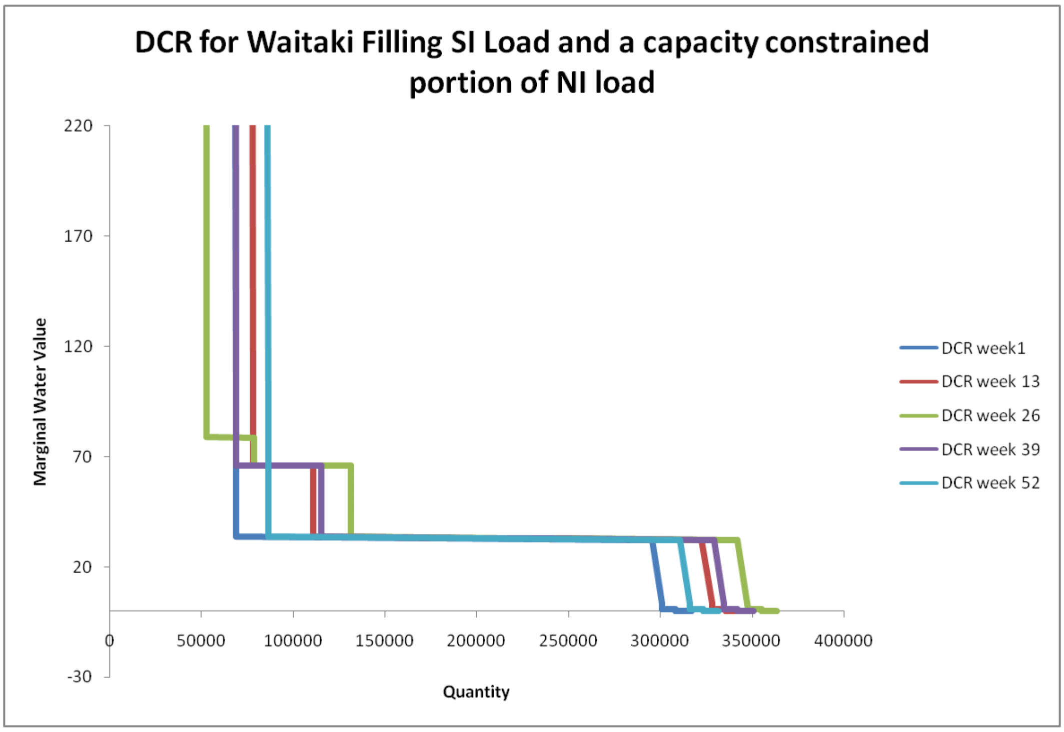


Figure 49 DCR for Waitaki Filling SI Load and a capacity constrained portion of NI load. Shows detail of Figure 40.

13. Two Reservoir Constructive Dual Dynamic Programming

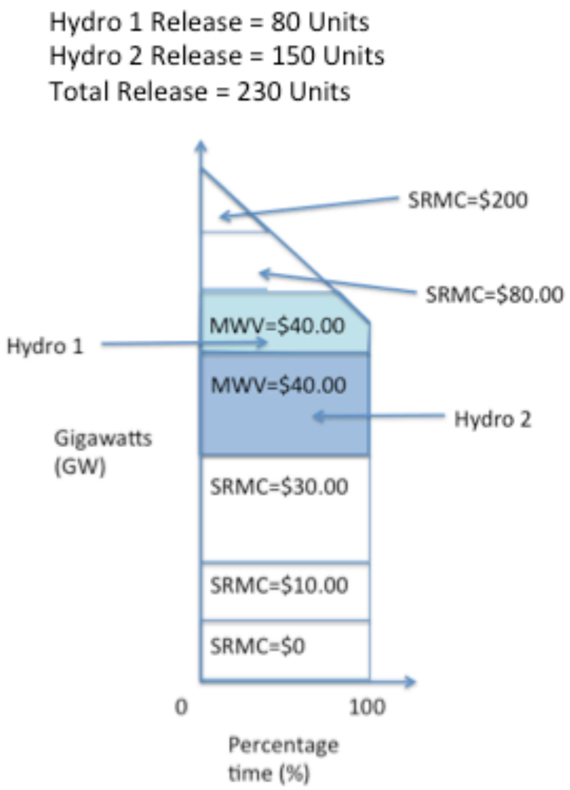
The extension of CDDP to two reservoirs is well-established in Winter (1987). In this section we will briefly overview some of the primary features the extension of CDDP generally to the case of two reservoirs which are in parallel in a single area. This will then be the basis for a discussion of how the point-wise algorithm discussed in R. A. Read et al. (2012) extends to the cornerwise SCDDP algorithm and applies to the case of two reservoirs in parallel in a single area. This will provide a basis for the extension of the SCDDP algorithm to two-reservoirs in parallel in a single area discussed in later sections, and then the application of the double filled LDC to this reservoir configuration.

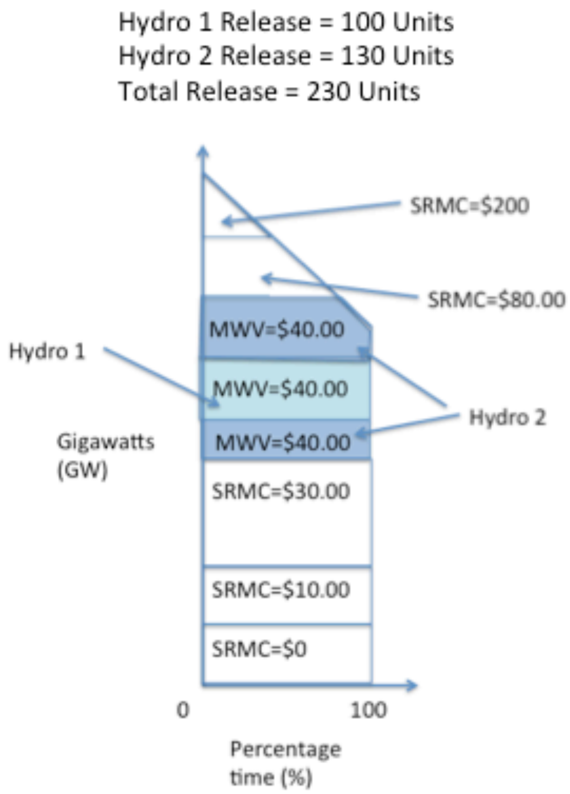
13.1 Formation of the DSR

To understand the impact of two-reservoirs in parallel in the same system, the first consideration must be the impact on the releases for one of these reservoirs as from this information a DSR can be formed. In a single reservoir system there are two possible key release levels for that reservoir with respect to each thermal. The maximum release at that MWV corresponds to the reservoir release completely displacing thermal generation with that SRMC in the merit order of filling the LDC. The minimum release at that MWV corresponds to the thermal generation completely displacing the reservoir release in the merit order of filling the LDC. When another reservoir is also supplying load to fill the same LDC then the complexity of the problem increases. There are now four possibilities:

- The reservoir is higher in the merit order than both thermal at the MWV and other hydro generation
- The reservoir is higher in the merit order than only thermal generation at that MWV
- The reservoir is higher in the merit order than only hydro generation at that MWV
- The reservoir is lower in the merit order than both thermal and hydro generation at that MWV

Ignoring transmission losses and assuming constant generation efficiency from each reservoir, storage and release can be measured in energy terms, and here we have assumed the critical MWV ratio between the two reservoirs 1. For higher ratios the minimum DSR will apply for one reservoir and the maximum for the other. For lower ratios, the situation will be reversed. At the critical ratio, we are indifferent between release from the two reservoirs, and a range of intermediate solutions may be optimal, provided the same total energy is produced. All four of the configuration options are displayed below.





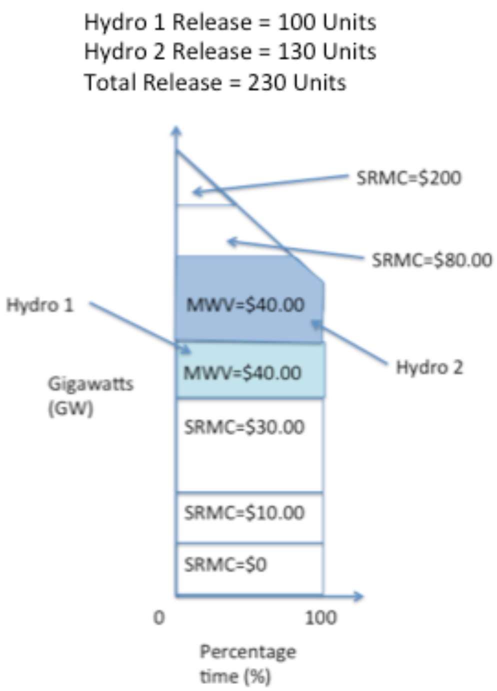


Figure 50 Possible configurations of two reservoirs in series filling the LDC

Note that the release from Hydro 1 will remain the same when Hydro 2 is below Hydro 1 in the merit order irrespective of how far below the position of Hydro 2 actually is. Likewise where Hydro 2 is above Hydro 1 in the merit order then the release from Hydro 1 will remain constant irrespective of the difference in the position of Hydro 2.

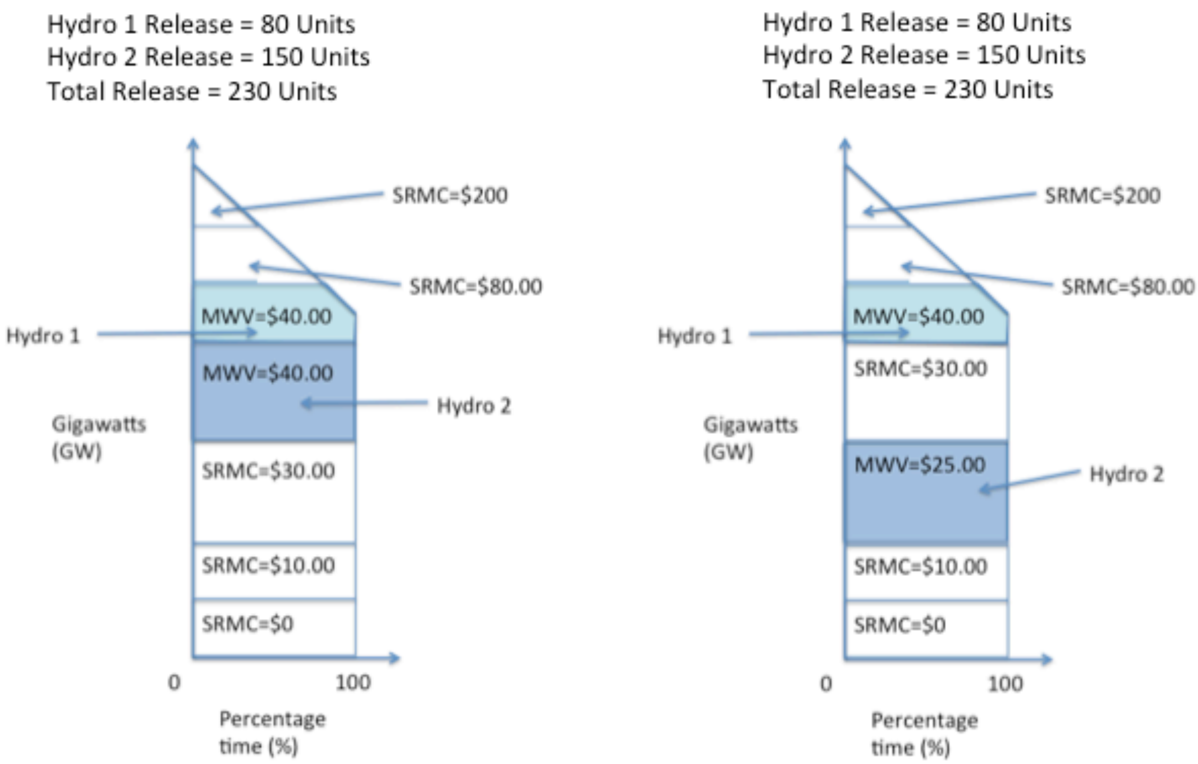


Figure 51 Location of Hydro 2 does not impact Hydro 1 release provided Hydro 2 remains higher in the merit order

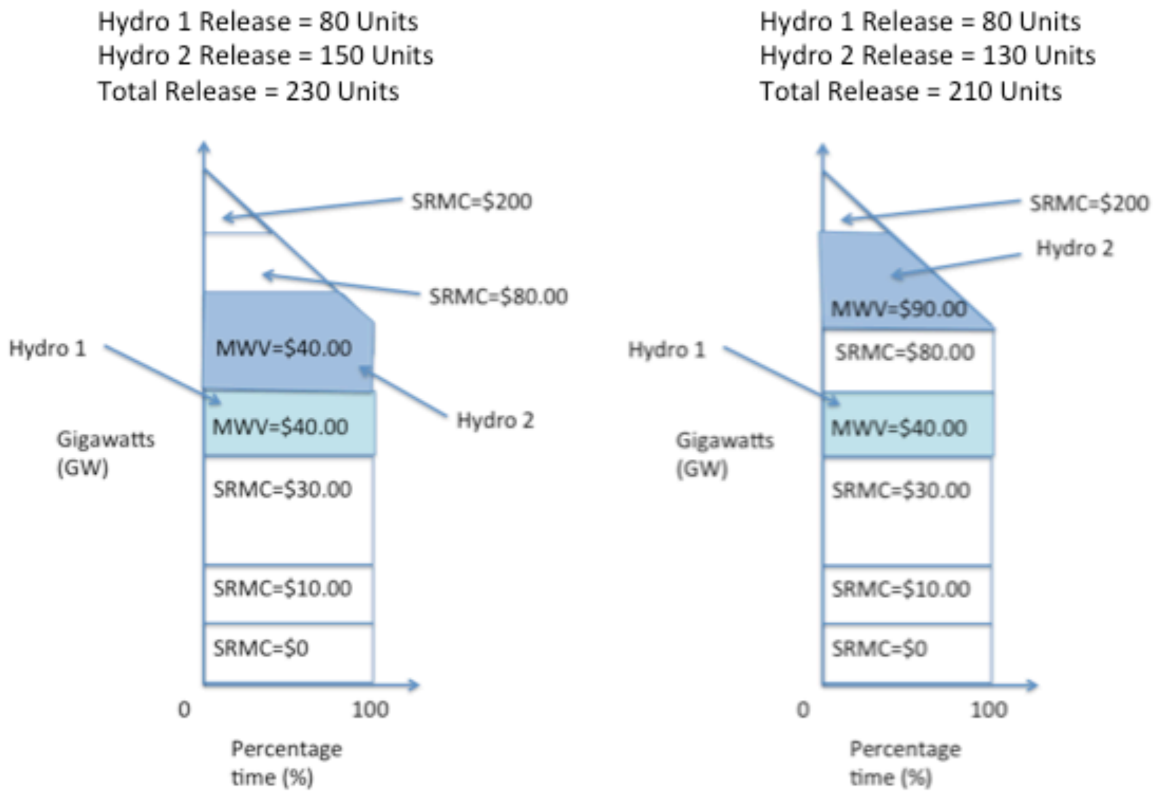


Figure 52 Location of Hydro 2 does not impact Hydro 1 release provided Hydro 2 remains lower in the merit order

To represent the DSR for the single reservoir case, only the position of the hydro-electric reservoir relative to the thermal at a particular MWV was recorded. To represent the DSR for the two-reservoir case, both then the relative position of the two reservoirs, and the position of the hydro-electric reservoir relative to the thermal at the MWV must be recorded. That is to say, where Hydro 1 has a fixed MWV, then there exists a minimum and maximum release level for Hydro 1 at that MWV where Hydro 2 is higher in the merit order of filling the LDC than Hydro 1. Likewise where Hydro 2 is lower in the merit order of filling the LDC than Hydro 1 there is a different minimum and maximum release level for Hydro 1 at that MWV. These are shown in the table below.

Maximum Release Hydro 1	Minimum Release Hydro 1
80	60

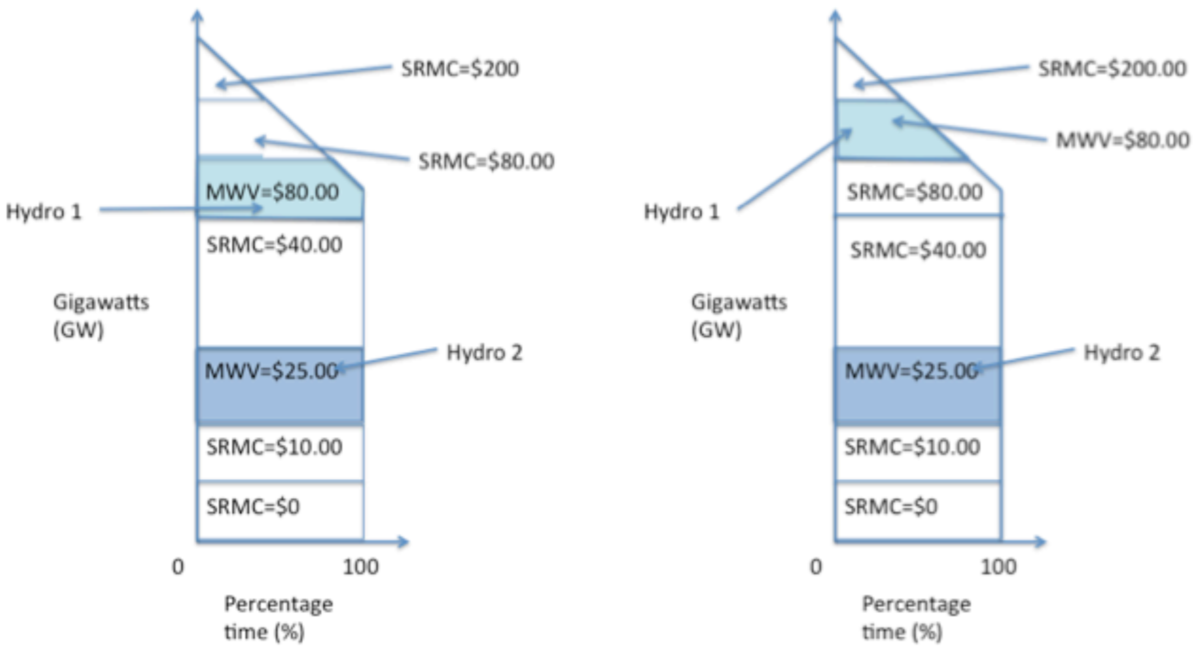


Figure 53 Merit order swap between Hydro 2 and thermal with SRMC = $80.00

Maximum Release Hydro 1 100

Minimum Release Hydro 1 90

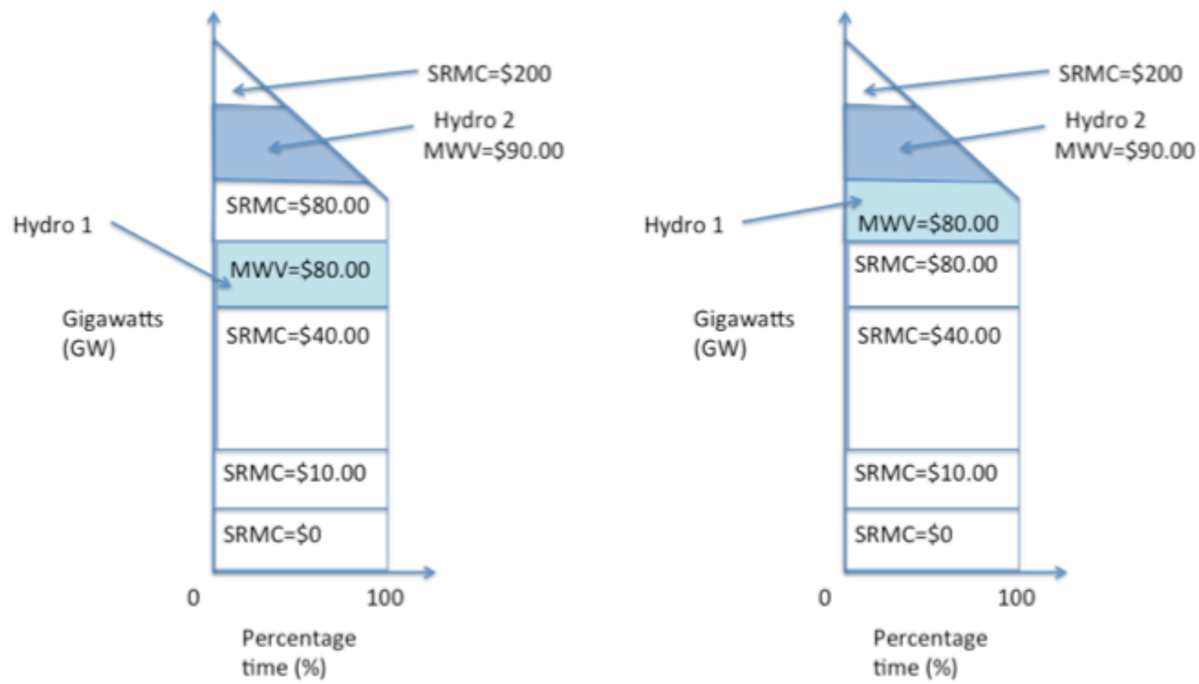


Figure 54 Merit order swap between Hydro 1 and thermal with SRMC = $80.00

	Maximum Release MWV=$40	Minimum Release MWV=$40
Hydro 1 above Hydro 2 in Merit Order	80	60
Hydro 2 below Hydro 1 in Merit Order	100	90

Table 1: Maximum and Minimum releases for Hydro 1 where a merit order swap occurs

Hydro 2 will also have a corresponding set of two maximum and minimum release pairs at that MWV that are dependent on the position of Hydro 2 relative to Hydro 1. Consider the situation where both hydro-reservoirs are above the thermal generation in the merit order. As Hydro 1 displaces Hydro 2 then the release level from Hydro 1 will move from Hydro 1’s maximum release given Hydro 2 is higher in the merit order to Hydro 1’s maximum release given Hydro 2 is lower in the merit order. Simultaneously the release level from Hydro 2 will move from Hydro 2's maximum release given that Hydro 1 is lower in the merit order to Hydro 2's maximum release given Hydro 1 is higher in the merit order. This shift is shown both in terms of the above tables and the above LDCs below.

Maximum Release Hydro 1	Maximum Release Hydro 1
100	80
Release Hydro 2	Release Hydro 2
110	130

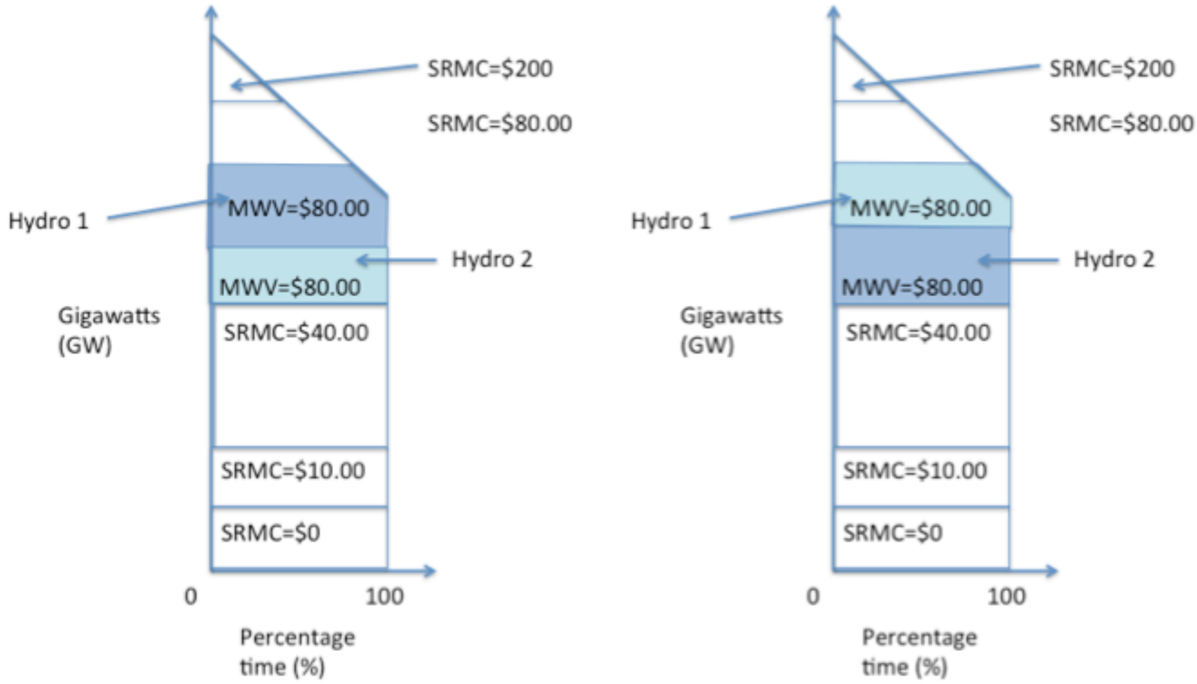


Figure 55 Merit order swap between Hydro 1 and Hydro 2

If we look at this graphically for Hydro 1 in terms of both reservoir's respective release levels then this produces a 'guideline' to delineate all possible optimal release combinations from Hydro1 as Hydro 2 is displaced in the merit order where Hydro 1 is above the thermal in the merit order of filling the LDC. This guideline represents all possible release combinations that produce optimal utility in reducing system costs. Note that the horizontal components of the guideline represents that the release solution stays the same irrespective of how far above or below Hydro 1 that Hydro 2 is in the merit order.

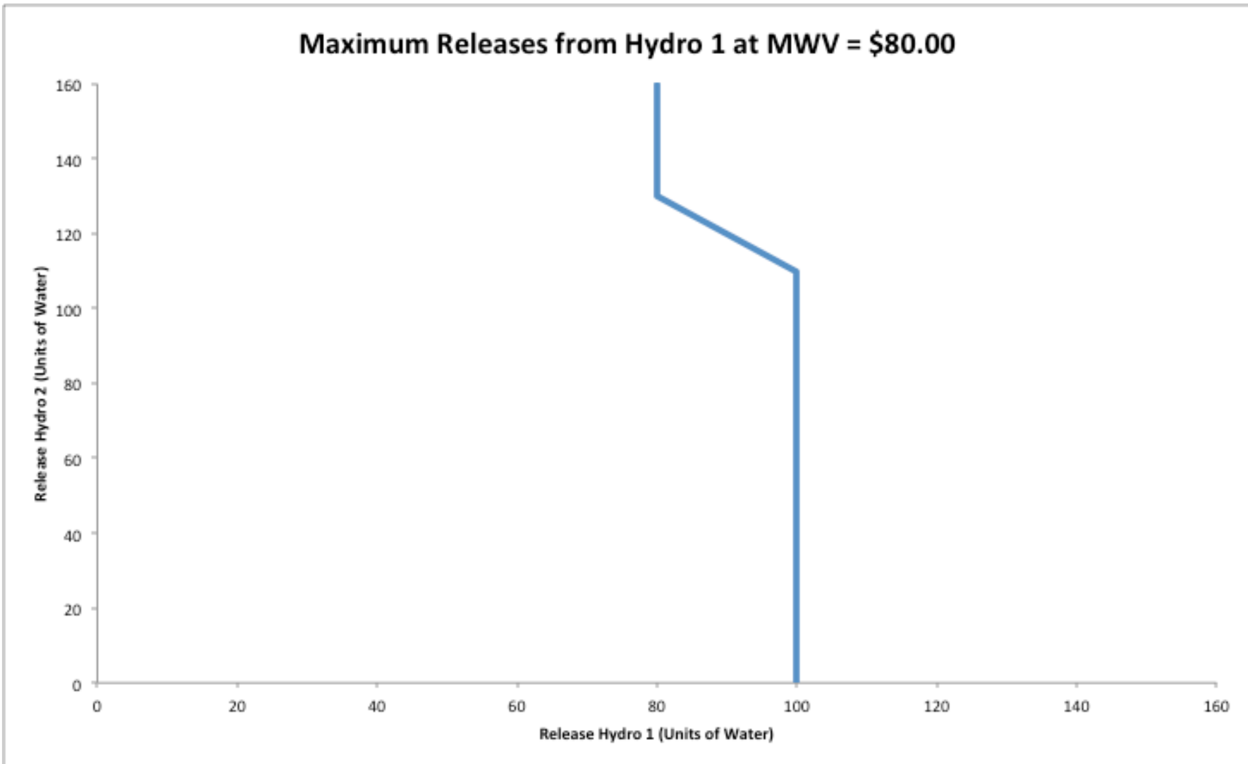


Figure 56 Guideline of maximum release from Hydro 1 at MWV = $80.00

A corresponding shift occurs where Hydro 1 displaces Hydro 2 where both reservoirs are below the thermal in the merit order. The additional guideline in the graphical representation again maps optimal release combinations where the MWV of Hydro 1 remains constant, however for this guideline both reservoirs are below the thermal in the merit order.

Minimum Release Hydro 1	Minimum Release Hydro 1
90	60
Release Hydro 2	Release Hydro 2
90	120

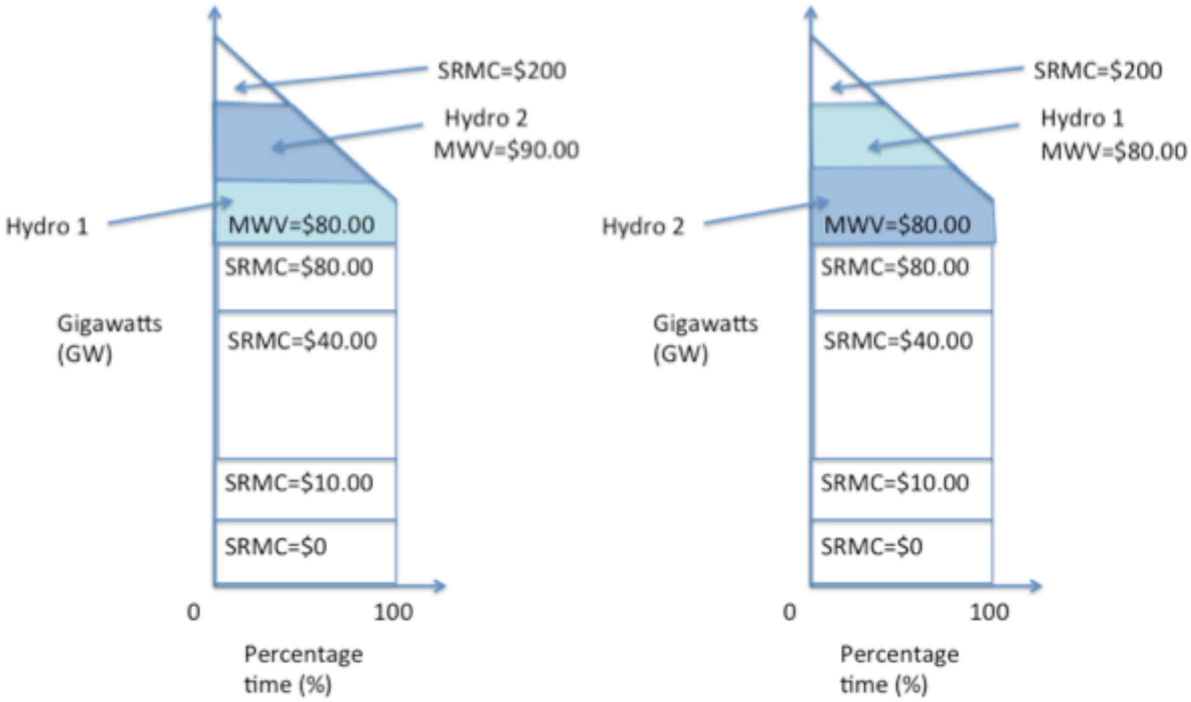


Figure 57 Merit order swap between Hydro 1 and Hydro 2

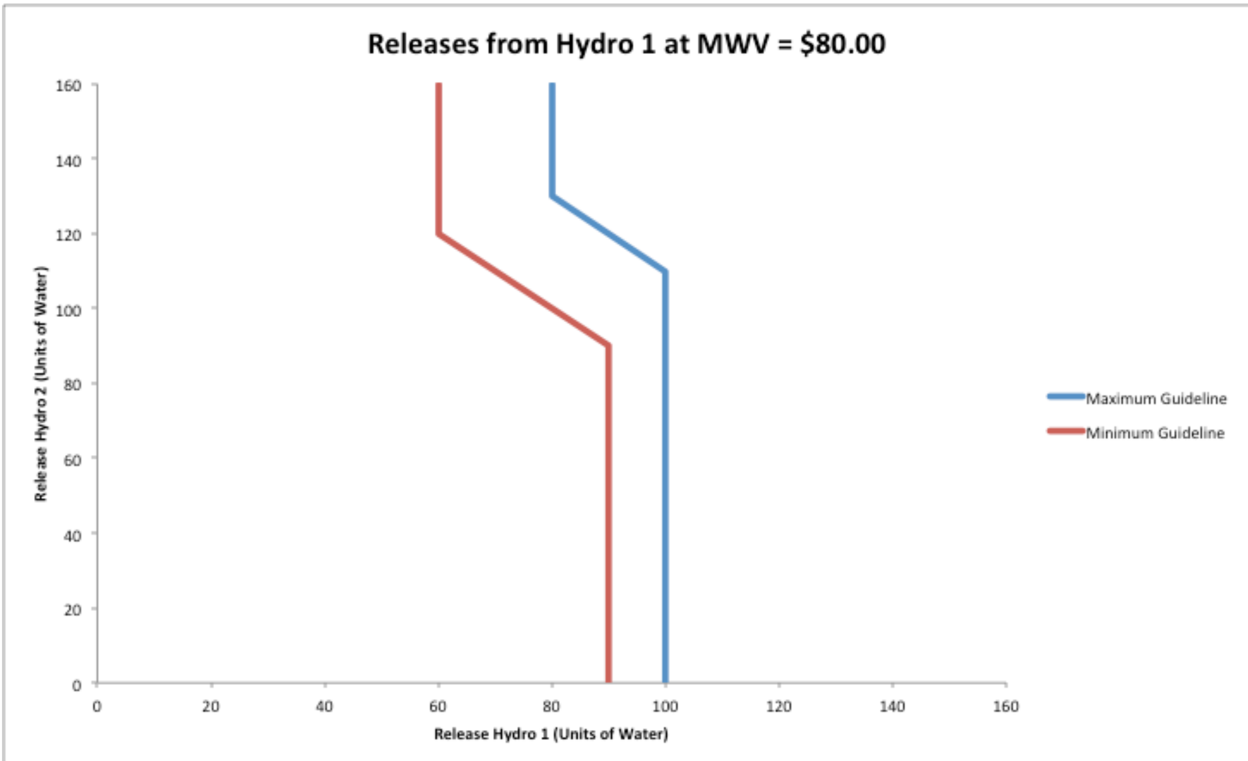


Figure 58 Maximum and minimum release guidelines for Hydro 1 at MWV = $80.00

In the below diagram then the same trade off occurs except the MWV of Hydro 2 is held constant throughout and the optimal releases from Hydro 2 are mapped.

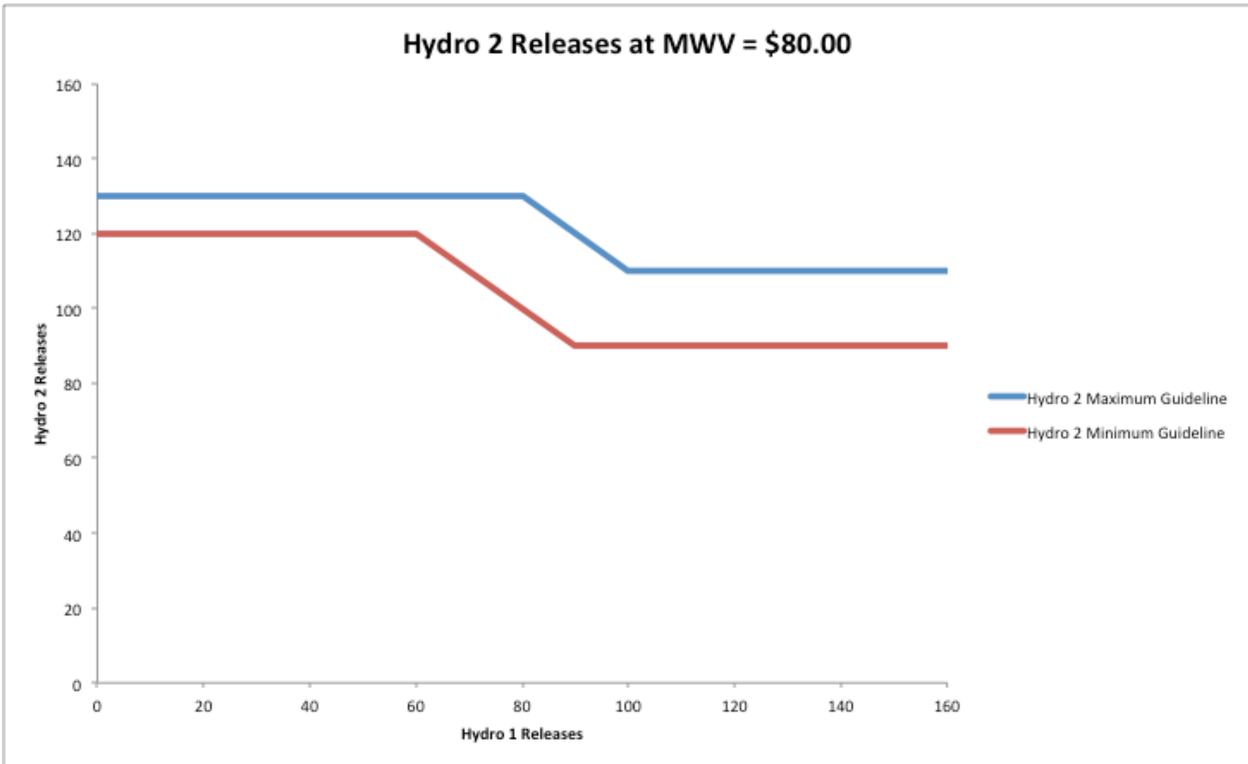


Figure 59 Maximum and minimum release guidelines for Hydro 2 at MWV = $80.00

As can be seen below, these two diagonal areas of indifference are equivalent whether considered from the perspective of Hydro 1's MWV being held constant, of the MWV of Hydro 2 being held constant. This reflects the fact that in either circumstance both MWVs are equal.

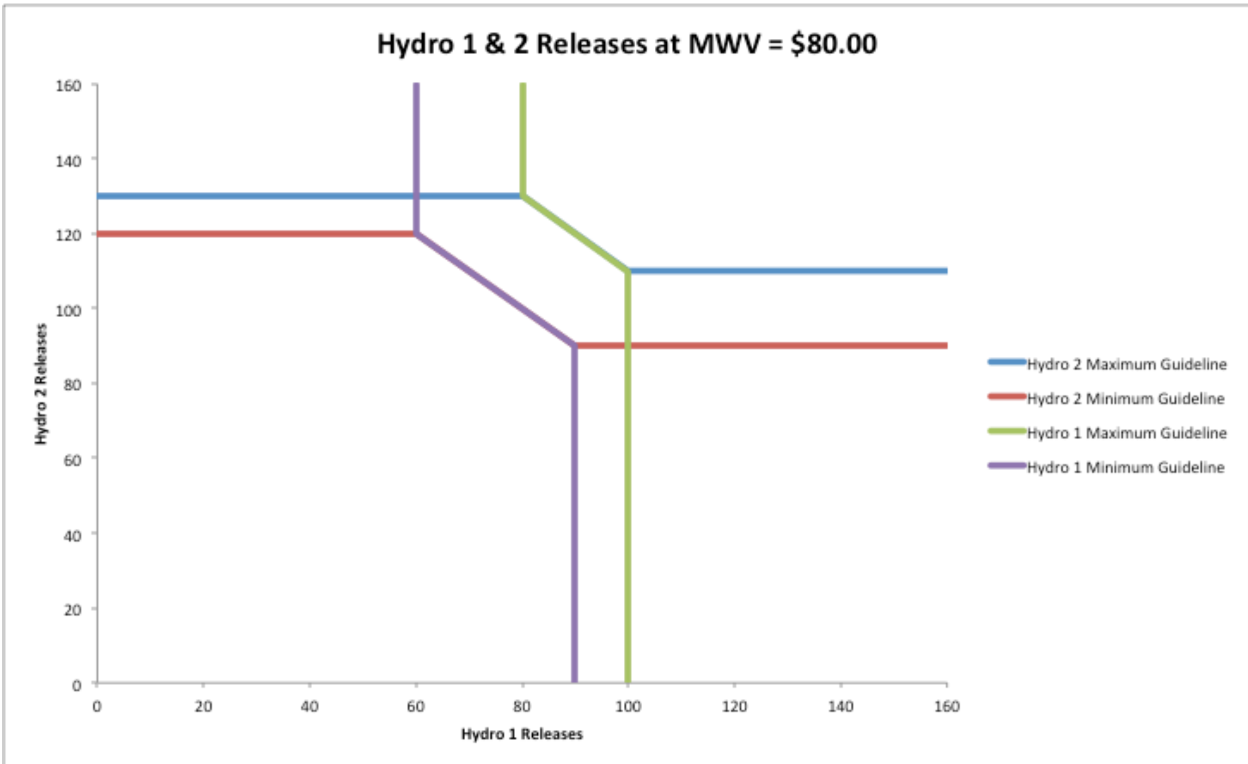


Figure 60 Hydro 1 and Hydro 2 release guidelines at MWV = $80.00

On the below set of diagrams the key shift of the reservoir Hydro 1 with respect to the thermal is identified in the single reservoir representation of the DSR, and the corresponding shift is identified on the two reservoir release diagram established above.

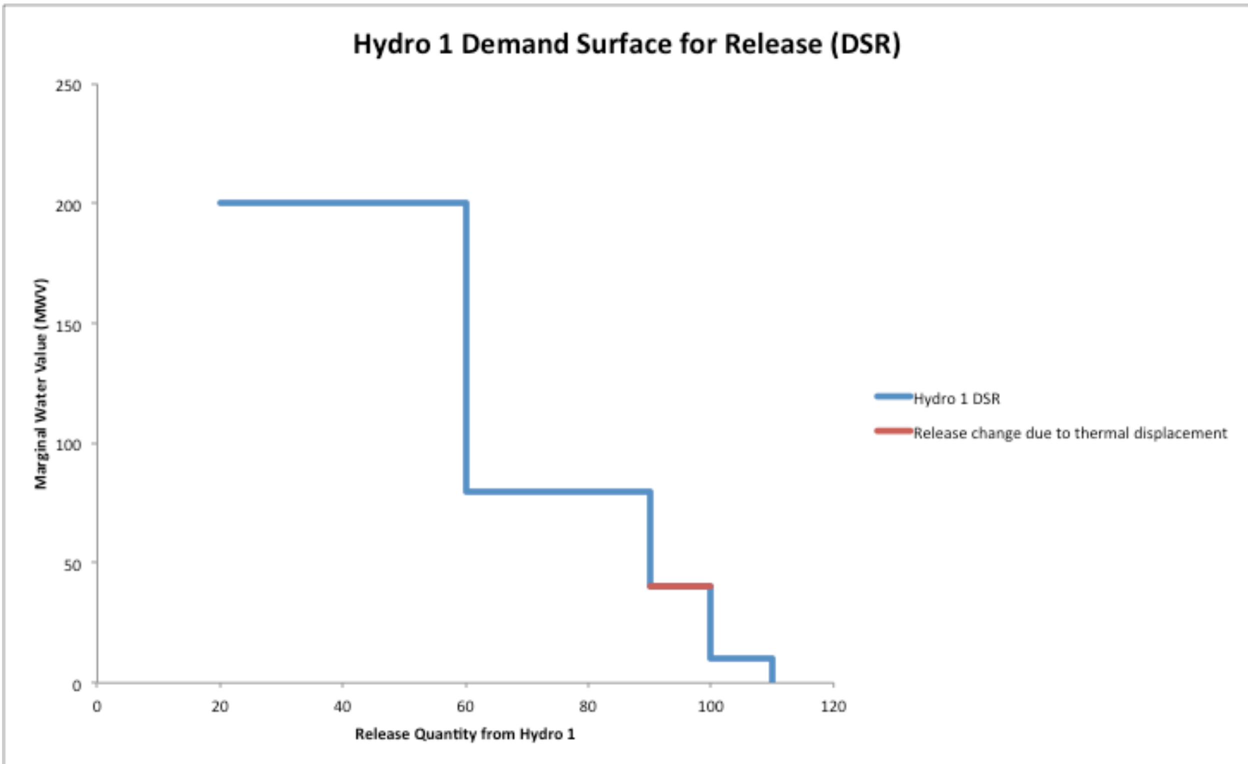


Figure 61 Hydro 1 Demand Surface for release

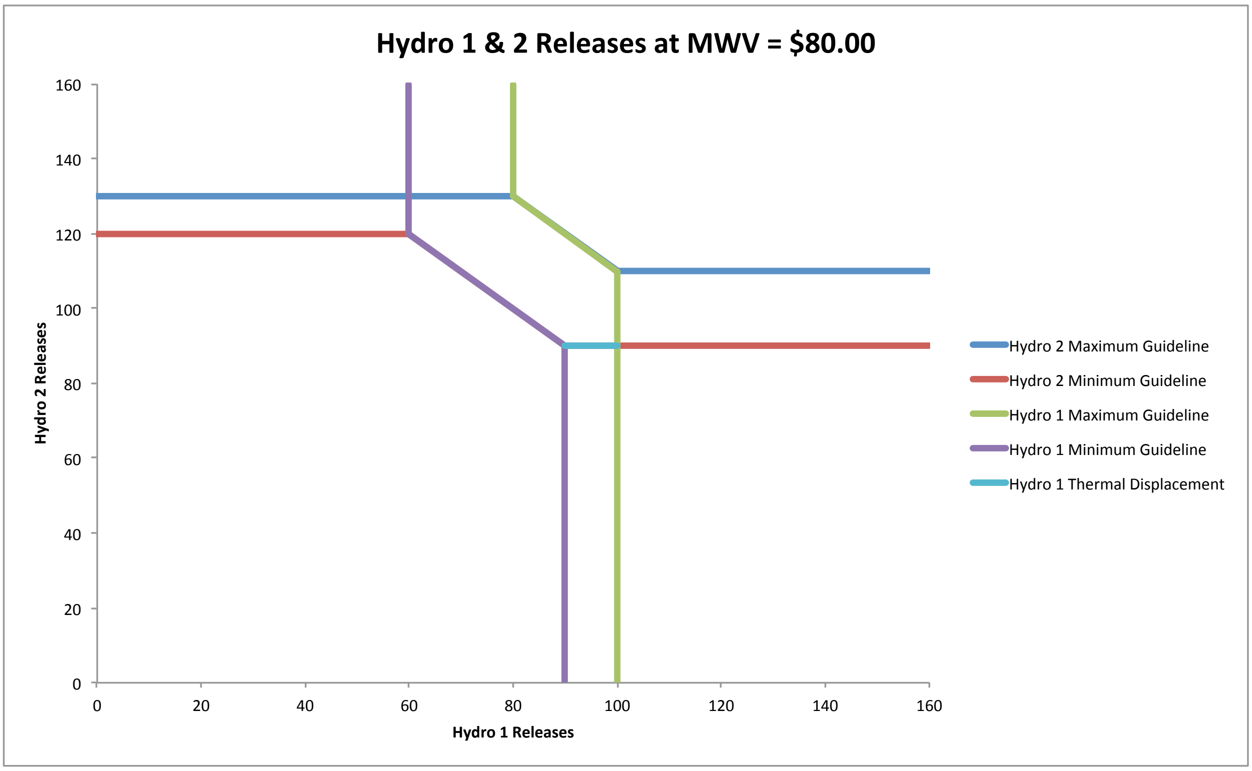


Figure 62 Hydro 1 thermal displacement is equivalent to release change due to thermal displacement in Figure 61

In the same way as for the single reservoir DSR, then the maximum release level at a given MWV is the same release level as the minimum release level for the next lowest critical MWV. This is the next lowest MWV at which the reservoir displaces the load in filling the LDC. Likewise the minimum release level at a given MWV is the same release level as the maximum release level for the next highest critical MWV. Consequently in practice the release diagram above will be graphically represented as shown below.

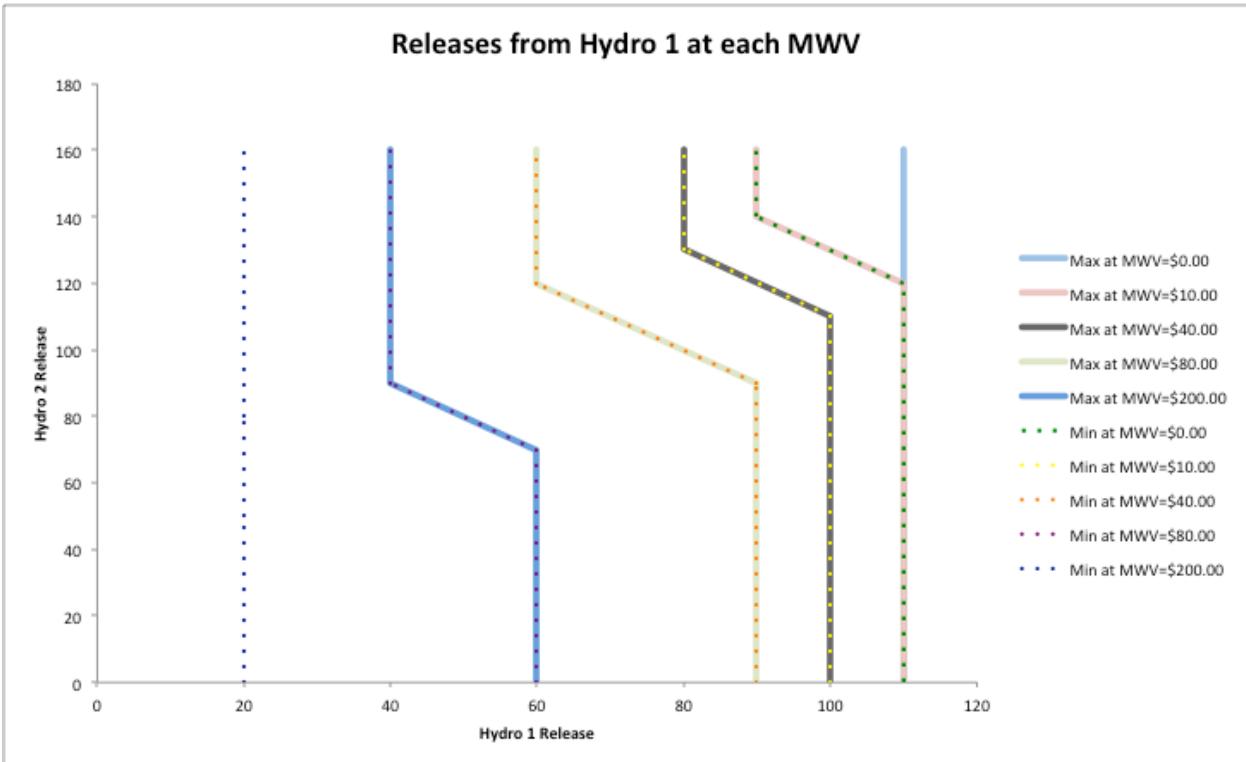


Figure 63 Total set of Hydro 1 release guidelines for all critical MWVs

This is effectively a contour plot of the DSR for Hydro 1. Given this diagram and that of thermal displacement in Figure 61 above at a particular MWV. The impact of the relative position of the other reservoir is shown in the guidelines that are established, while the impact of displacing the thermal at that MWV is a horizontal shift for Hydro 1. An equivalent diagram can also be created for Hydro 2 is also displayed below.

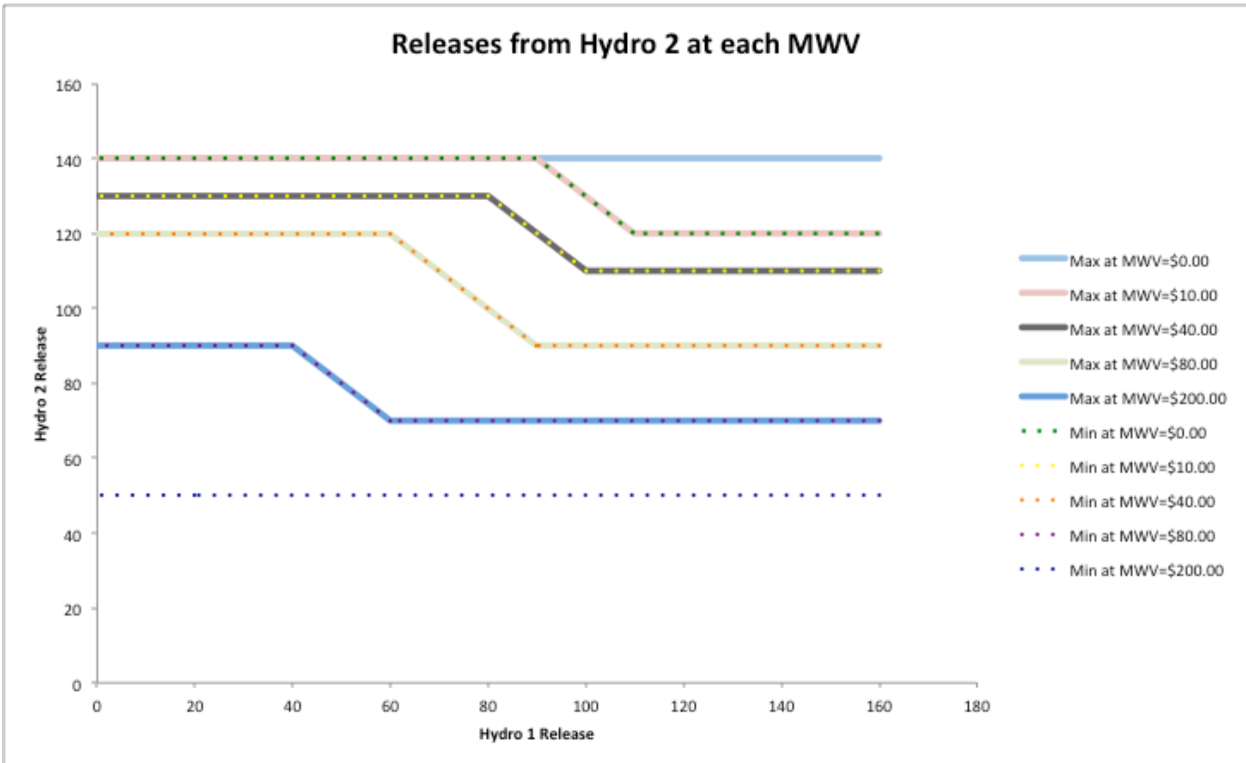


Figure 64 Total set of Hydro 2 release guidelines for all critical MWVs

To effectively represent the DSR, therefore, for Hydro 1, the graphical representation above could also be considered in terms of the table below. Assuming Hydro 2 remains constantly at the top of the merit order, Figure 65 ilustrates how release from Hydro 1 will decrease when the MWV of Hydro 1 rises above the marginal cost of a particular thermal generator. Each of these changes must be recorded. This is the equivalent of the two right---most columns in Table 2 below, where it is assumed that Hydro 1 is always below Hydro 2 in the merit order. These values are also represented again in the two middle columns of

Marginal Water Value	Hydro 1 MWV > Hydro 2 MWV		Hydro 1 MWV < Hydro 2 MWV	
	Maximum	Minimum	Maximum	Minimum
$0.00	110	90	110	110
$10.00	90	80	110	100
$40.00	80	60	100	90
$80.00	60	40	90	60
$200.00	40	20	60	20

Table 3. These release quantities represent the maximum and minimum release associated with displacing the thermal generator. At this MWV any combination of the two sources that produces the same total generation has the same cost, and any intermediate release level can be optimal. Thus, increasing MWV from zero to

infinity so that reservoir release starts at the bottom of the LDC and moves up to the top of the LDC, produces a monotone decreasing component of the DSR, consisting of a series of quantity steps. These are the vertical components in the above graphical representation.

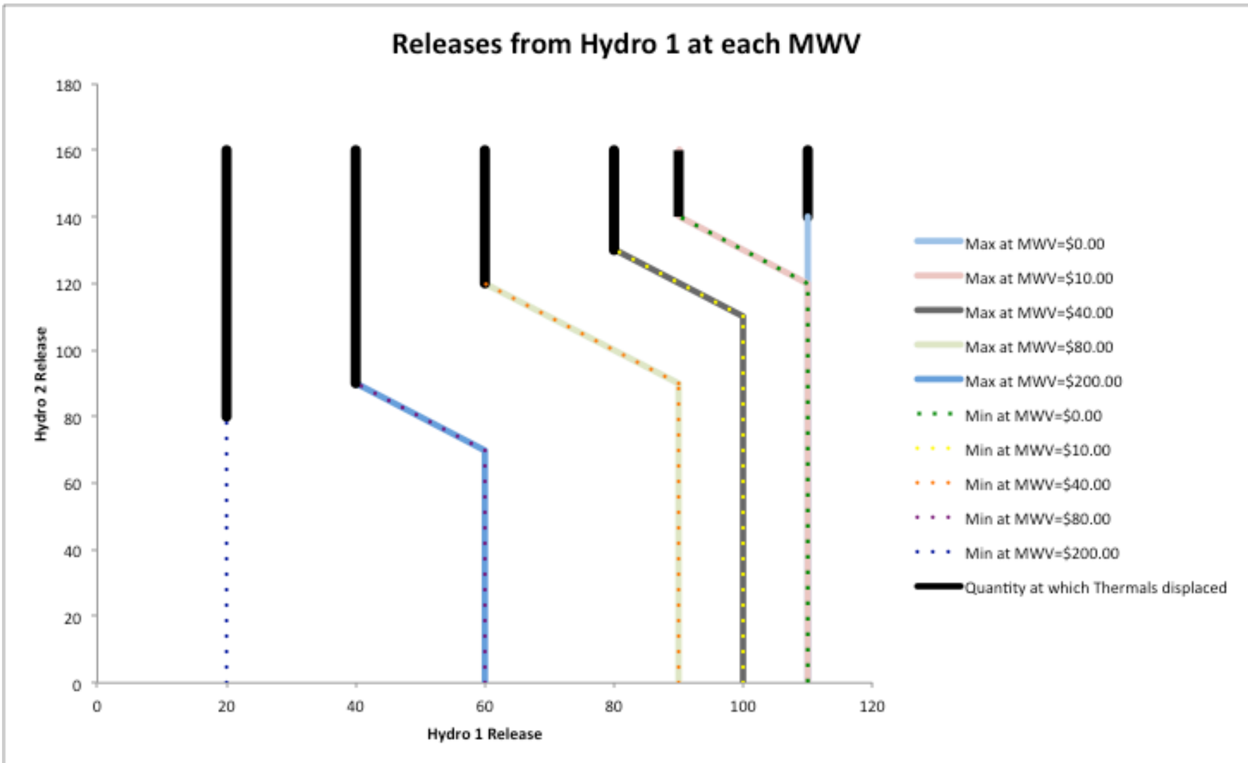


Figure 65 Thermal displacement component of Hydro 1 release guidelines where Hydro 1 MWV > Hydro 2 MWV

Marginal Water Value	Maximum	Minimum
$0.00	110	90
$10.00	90	80
$40.00	80	60
$80.00	60	40
$200.00	40	20

Table 2 Release quantities from Hydro 1 at each MWV where Hydro 1 MWV > Hydro 2 MWV

Hydro 1's DSR can then be completed by increasing the MWV of Hydro 1 from zero to infinity while assuming that Hydro 2 is always below Hydro 1 in the merit order. This provides the two right-most columns of the table below. The graphical component in the release diagram is likewise both shown below.

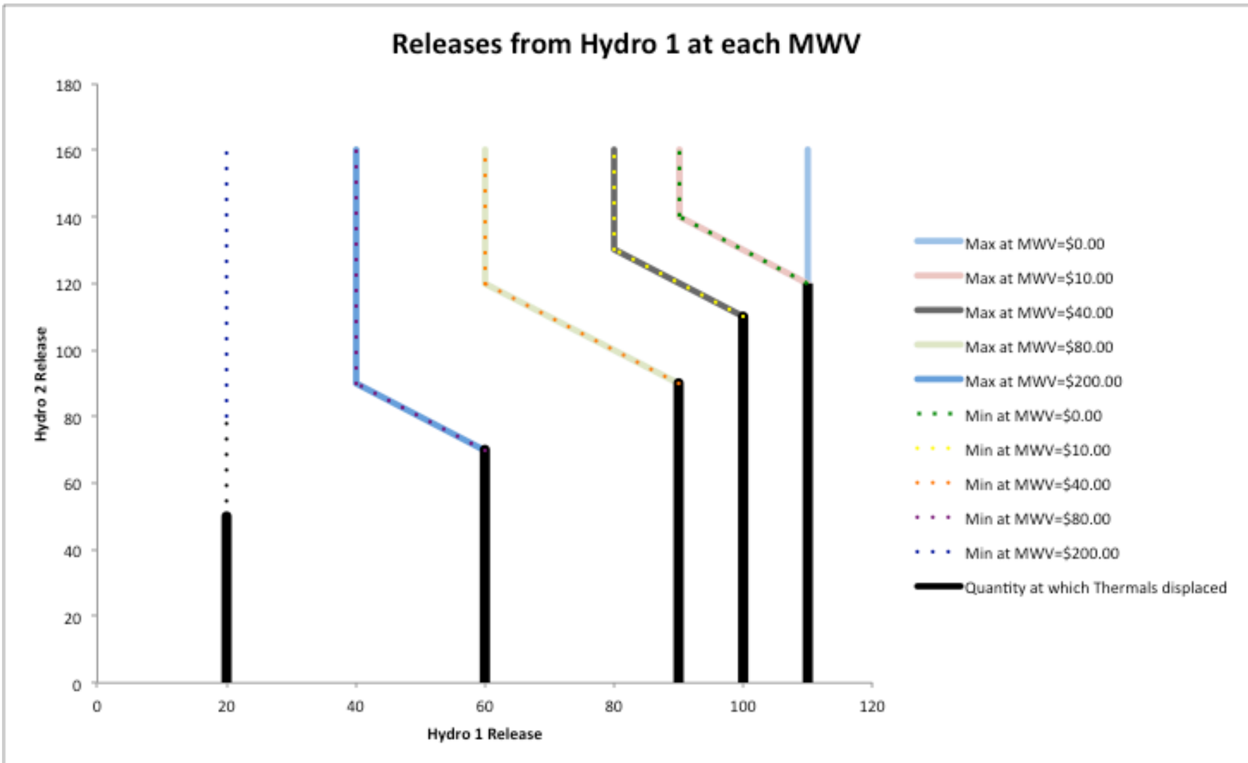


Figure 66 Thermal displacement component of Hydro 1 release guidelines where Hydro 1 MWV < Hydro 2 MWV

Marginal Water Value	Hydro 1 MWV > Hydro 2 MWV		Hydro 1 MWV < Hydro 2 MWV	
	Maximum	Minimum	Maximum	Minimum
$0.00	110	90	110	110
$10.00	90	80	110	100
$40.00	80	60	100	90
$80.00	60	40	90	60
$200.00	40	20	60	20

Table 3 Release quantity limits from Hydro 1 at each MWV given the relative position of Hydro 2

On the graphical representation of releases below the shaded area is the area in which the MWVs for both reservoirs are at the critical ratio, and an area of indifference occurs. The diagonal line segments crossing this area are common to the guidelines for both reservoirs. Outside that zone, all release guidelines are horizontal for Hydro 2 and vertical for Hydro 1 because the LDC area covered by release from each reservoir, does not change as the MWV of the other reservoir moves further away from the critical ratio.

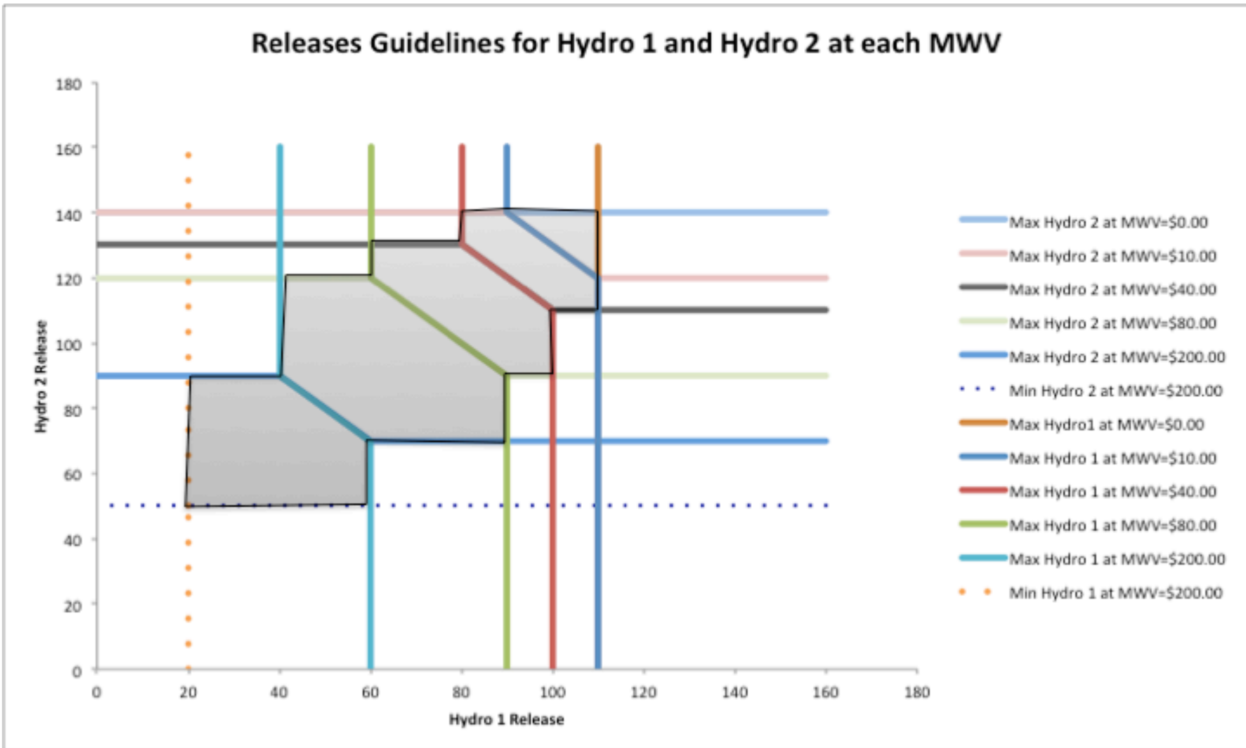


Figure 67 Area of indifference where reservoirs MWVs are at critical ratio

As described for the single reservoir algorithm in order to allow the CDDP algorithm to be more readily generalisable to higher dimensionality, for the cornerwise algorithm DSRs are preR computed and representations are stored in a more compact form than in previous CDDP implementations. Here we only compute release and storage levels for a discrete set of critical MWVs, and MWV combinations.

The maximum release values are also the minimum values of the next highest critical MWV **as** seen on

Marginal Water Value	Hydro 1 MWV > Hydro 2 MWV		Hydro 1 MWV < Hydro 2 MWV	
	Maximum	Minimum	Maximum	Minimum
$0.00	110	90	110	110
$10.00	90	80	110	100
$40.00	80	60	100	90
$80.00	60	40	90	60
$200.00	40	20	60	20

Table 3 above. Hence the two DSRs (one for each reservoir) can be entirely defined by the list of critical MWVs, and the corresponding set of points delineating each corner point on Figure 63 and Figure 64 above. From these points and the guidelines for each reservoir may be inferred. This holds true for set of parallel reservoirs. However, where there are intervening transmission losses or reservoirs are in series it may be necessary to expand this representation.

13.2 CDDP Algorithm

DSR formation establishes the form of the inputs to the CDDP algorithm. This cornerwise delineation is then mirrored throughout the algorithm process. In particular, the DSS is formed by the addition of inflow-adjusted DSRs. Hence where all DSRs are represented in terms of the same critical MWVs then the DSS can likewise be represented compactly. The DSS is then also truncated to reflect reservoir limits this can produce an exception to that generic representation.

As is established for the single reservoir CDDP case above, there is a DSS for each period for each reservoir. Working backwards recursively from the end of the planning horizon forms this DSS. For two reservoirs in parallel the critical MWVs at which the DSS solution changes are those that are recorded for DSR storage above. This is effectively a master list of all critical MWV levels and ratios that may occur over the planning horizon. The storage surface between these points is implicit in this construction. If there is no known future DSR value for releasing that unit in the future then that unit will have no impact on the release or storage. If there is a known future DSR value then that will be implicitly included in the construction of the DSS from future DSRs. Ignoring wastage, holding costs and discounting, no other MWV levels can actually occur in the deterministic case. Every unit of water will eventually be used to fill load at one of these critical marginal cost levels. Hence rather than specify which MWV level that will be for an arbitrary set of storage points, the CDDP algorithm determines the set of storage pairs that forms the bounds over which each possible critical MWV level applies where the other reservoir is higher in the merit order than the reservoir the DSS is for.

The CDDP algorithm also determines the set of storage pairs that form the bounds over which each possible critical MWV level applies where the other reservoir is lower in the merit order than the reservoir the DSS is for.

To form these DSS surfaces the algorithm simply determines the storage pair corresponding to each of these critical price pairs, recursively, for each period. Both maximum and minimum levels of water at a given price are necessary to produce an accurate representation of the DSS. This determination is performed by the cornerwise addition of the DSRs formed above. The critical corners described here are the two reservoir equivalent of the critical points discussed in the single reservoir CDDP description above. As the reservoir numbers increased to three reservoirs these would correspondingly become critical edges.

$Where\ t = Time, v_1 = Critical\ marginal\ water\ value\ res.1$

$v_2 = Critical\ marginal\ water\ res.2, r = res, r_1 = res$

$$DSS'_{t,r,v_1,v_2} = DSS_{t,r,v_1,v_2} + DSR_{t,r,v_1,v_2}$$

$$DSS_{t-1,r,v_1,v_2} = \left(DSS'_{t,r,v_1,v_2} - inflow_{t,r}\right) \quad \forall t, \forall v_1, \forall v_2$$

$$if\left(DSS'_{t,r,v_1,v_2} - inflow_{t,r}\right) < Smin_r$$

$$truncate\ DSS_{t-1,r,v_1,v_2} \forall v_1\ to\ reflect\ Smin_r$$

$$if\left(DSS'_{t,r,v_1,v_2} - inflow_{t,r}\right) > Smax_r$$

$$truncate\ DSS_{t-1,r,v_1,v_2} \forall v_1\ to\ reflect\ Smax_r$$

Algorithm 10

Applying the above algorithm will first produce a set of critical interim water volume pairs, which define the interim DSS' state over an expanded storage range. This interim DSS' is then offset by the expected inflow pair to form the beginning of period DSS. However, this DSS will be truncated to reflect reservoir limits when it is passed through as an input to the previous period sub-problem. These reservoir limits truncate the DSS where the DSS includes points outside the feasible storage range. These solutions either imply the storage of water for future use which cannot be physically held in the reservoir, or else drawing down water for use in the current time period which has not yet arrived in the reservoir. In order to limit the solution space to feasible solutions the interim DSS must be truncated in order to take into account the reality of the storage levels. In the two-reservoir case, however, there is a requirement for care to be taken in ensuring that the truncation method preserves both MWVs accurately on the bounds.

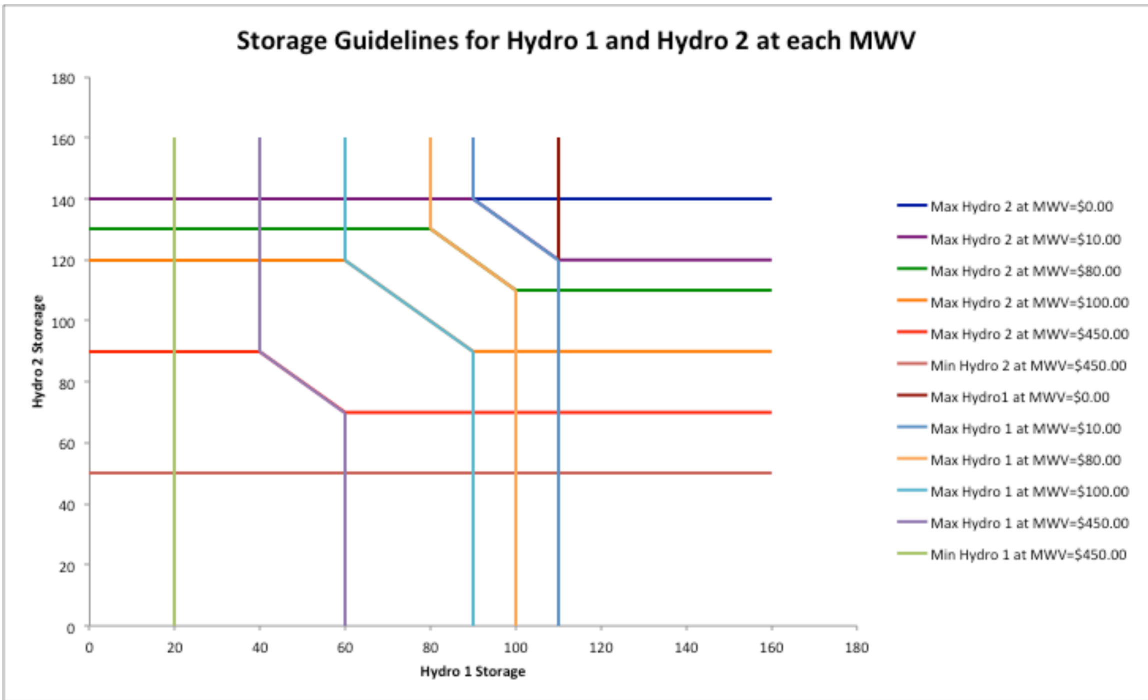


Figure 68: Storage Diagram two reservoir prior to truncation

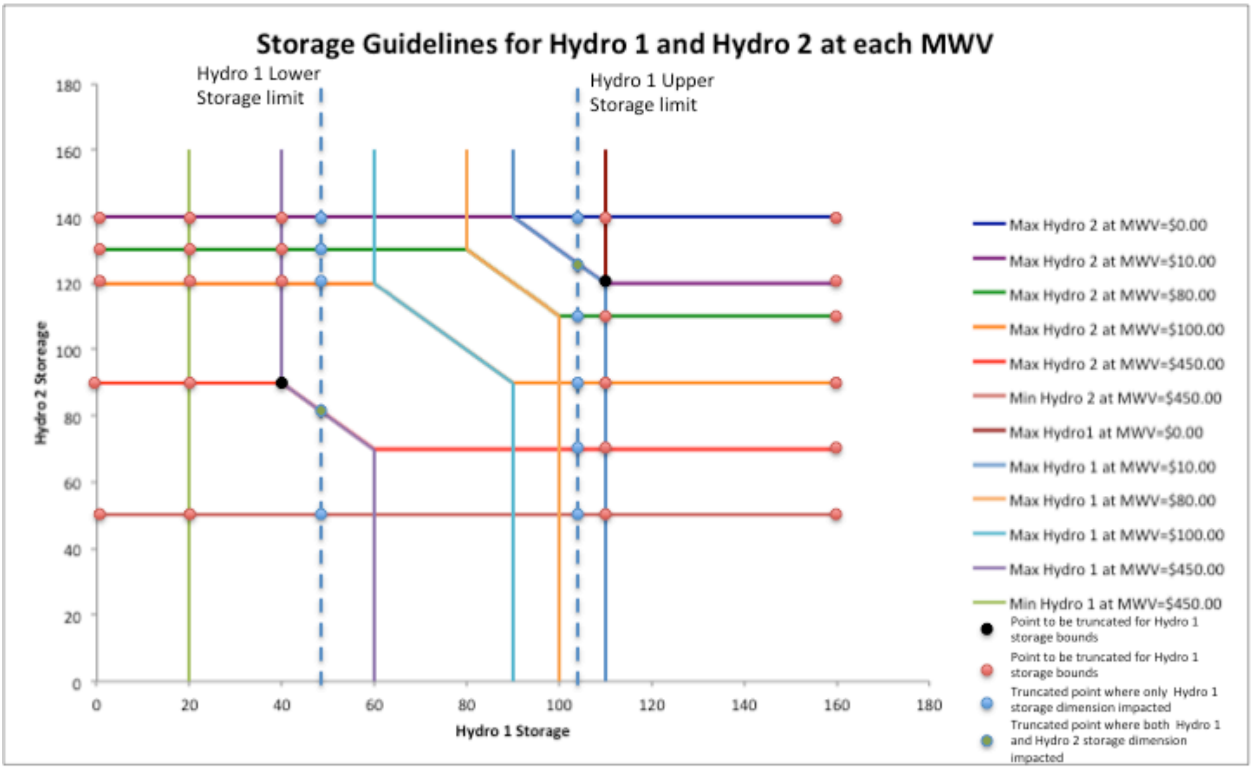


Figure 69: Storage Diagram two reservoir Hydro 1 truncation

Three different possible implications of truncation for reservoir limits in the storage space are displayed in the diagram above. The first and simplest of these implications is for the sections where the storage limit intersects the horizontal component of Hydro 2 Storage guidelines. At this Hydro 1 Storage level the Hydro 2 storage level is constant and independent of the Hydro 1 Storage level. Where this scenario occurs then all critical points recorded along this horizontal guideline are truncated such that the Hydro 2 Storage level remains the same and the Hydro 1 Storage level is equal to:

Max ($s_{hydro\ 1}$, Min $Storage_{Hydro\ 1}$)

where the horizontal section of the Hydro 2 guideline intersects the minimum storage bound for Hydro 1 .

Min($s_{hydro\ 1}$, Max $Storage_{Hydro1}$)

Where the horizontal section of the Hydro 2 guideline intersects the maximum storage bound for Hydro 1.

On Figure 69 above this is represented by the red points on the horizontal guidelines being forced to equal the blue points at which the horizontal guideline is intersected by the storage limit.

The second possible implication is for the sections where the entire Hydro 1 (vertical) guideline is outside of the storage bounds for Hydro 1. Where this occurs the Hydro 1 guideline for the MWV level is truncated to equal to most proximate storage limit that it is outside of. On Figure 69 above this is represented by the red dots on the vertical aspect of a guideline being truncated to equal the green dots that are found by moving horizontally across from this position.

The third possible implication is where the storage bounds intersect both the Hydro 1 (vertical) guideline and Hydro 2 (vertical) guideline at a point where a trade off is occurring in the solution space between storage in the respective reservoirs. In order for these guidelines to be truncated to reflect the Hydro 1 reservoir limits the storage of Hydro 1 be limited to the Hydro 1 storage bounds. This implies an increase or decrease in the level of Hydro 1 storage that is compensated for by a proportionate decrease or increase respectively in the level of Hydro 2 storage to ensure that the critical ratio between the two reservoirs at that MWV is preserved.

On Figure 69 above this is represented by the black dots on a guideline being truncated to equal the green dots on that guideline for both the Hydro 1 and Hydro 2 storage guidelines. The remaining vertical aspect of the Hydro 1 guideline will be truncated in accordance with the ‘second implication’ described above.

The storage limits for the Hydro 2 reservoir would be implemented in a corresponding fashion except inasmuch as they are horizontal reservoir limits rather than vertical reservoir limits on the diagram above.

Generally additional key points only arise where reservoir storage limitations prevent the storage, or use of certain water quantities which are within the region at which we are indifferent between two generation sources at this MWV. This is because where water cannot be stored in one reservoir to fill future load it may be supplemented by the storage of water in the other reservoir to fill this portion of load instead. Since the release or storage level at a given MWV is constant for any limitations of the other reservoir’s release or storage availability outside of this indifference region then it is unnecessary to store the truncation points outside of this region.

The truncation alters the shape of the total indifference region, and can also create multiple coincident points that may seem redundant. However the guidelines outside the indifference area are still formed by horizontal or vertical projection from these corner points. So the algorithm treats these truncated points just like any other points, carrying them back into earlier periods where the pattern of inflows and release opportunities may eventually allow them to be considered in the feasible storage area. It is essential at this stage that these redundant points accurately reflect the storage limits as this prevents the decision maker for storing water for future use where at some point before the use occurred those units of water would have to be spilled due to reservoir constraints. This information corresponds to a seasonal cycle where generation options are likely to be employed at some times of the year yet would not be contemplated at other times.

14. Two Reservoir Stochastic Dual Dynamic Programming

The impact of extending a cornerwise CDDP algorithm for a 2-reservoirs in parallel is at an elementary level identical to the extension in the single reservoir case above. There are three key areas that need to be addressed: appropriate critical points to represent uncertainty, the impact of different truncation policies, and the choice of key points to facilitate the addition of the DSR to the DSS. The initial choice of critical points, and the choice of truncation policies while important, is straightforward. The choice of key points however has additional complexity reflecting the additional complexity in the truncation policy for a two-reservoir system. However we will address each of these issues in turn before presenting the SCDDP algorithm for two reservoirs in its fullest form.

This two-reservoir extension of SCDDP likewise uses the critical point + ε and critical point – ε representation of the EDSS in order to capture the impact of stochastic inflows. In two reservoirs however, there are two sources of uncertainty. There is uncertainty as to the inflow level for both reservoirs. This is not particularly influential where the reservoirs are not adjacent in the merit order of filling the LDC. However where the two reservoirs are adjacent then there are a range of possible resultant storage values for Hydro 1, and a range of possible storage values for Hydro 2.

Fundamentally this issue of uncertainty only impacts on the forming of water storage surfaces, as we do not know with certainty the future value which will be accrued by that stored unit of water. To cope with these two uncertainties, it is necessary to extend the representation of the critical corners to reflect the 'critical point + ε' and 'critical point – ε' with respect to both Hydro1 and Hydro2’s storage levels. However, the choice of ε for each reservoir may be different as each reservoir may be subject to a different inflow distribution. For each hydro reservoir, this would extend the tabulated reservoir storage representation mirroring the form of the DSR established for two reservoirs, except in that the quantity of points would be doubled for the SCDDP storage representation as each critical corner would be represented by two points in the Hydro 1 storage space instead of the single point used for CDDP, and likewise in the Hydro 2 storage space.

In a comparable way to the single reservoir case, one of the key factors will be the choice of ε for each reservoir. However, the implications for higher reservoir SCDDP flows intuitively from the discussion above as the ε value for each reservoir inflow distribution should be considered in isolation from the other reservoir.

The inflow distribution means that unlike the two reservoir CDDP case described above where there are clearly delineated relationships, here the storage surface of one reservoir must always be assumed to have some impact on the storage surface of the other. Hence when truncation occurs to reflect the storage limits of that reservoir then this also has implications for the other reservoir. This is identical in nature to the CDDP case described for Figure 69 above.

- Where the two reservoirs have different EMWVs then this will at most simply involve recording an addition point to reflect the lack of continuity in the Hydro 2 storage dimension.
- Where the two EMWVs are equal and we are indifferent between the two reservoirs, then the inability of one reservoir store above/below a certain point has implications for the other reservoir's optimal storage:
    - Where lower reservoir storage limits hold then an increase in the other reservoir's storage cannot be supplemented by a corresponding decrease in that storage. Consequently the other reservoir is unable to increase its storage above a specified quantity to maintain this ratio.

- Where upper reservoir storage limits hold then a decrease in the other reservoir's storage cannot be supplemented by a corresponding increase in that storage. Consequently the other reservoir is unable to decrease its storage below a specified quantity to maintain this ratio.
- Notably the point at which this limitation is imposed is effectively another critical storage point for both reservoirs. This means that truncations for the other reservoir's limits are also points to include in determining the representation of the DSS for addition to the DSR.

For the most part the issues in determining the representation of the DSS and DSR in order to perform cornerwise addition are identical for both the single reservoir and two reservoir cases. For this reason the chosen representation is yet again the key DSR points, alongside sets of points to represent the truncation of the DSS surface. In considering the two reservoir as opposed to the one reservoir representation of the DSS for cornerwise addition there are two key differences; simplicity is of increased importance, and the truncation requires more accurate representation. Simplicity is of increased importance for the two reservoir case so that the compact representation of the EDSS is such that there are unlikely to be tractability issues even for a large number of time periods for two reservoirs. The compact cornerwise representation is also important as it produces a more generalisable structure for the extension of this algorithm beyond two reservoirs. The major aspect in allowing this generalisability is in identifying a consistent method of applying truncation for reservoir limits even when the problem cannot be readily visualised.

In truncating the guidelines that form the EDSS, two sets of truncation must be applied. The first of these is truncation for the storage limits of the reservoir with which the guidelines are associated. In every discernible respect this is the equivalent of the truncation in a single dimension and can be treated identically, with two points corresponding to the MWVs just either side of the critical truncation point being incorporated in the DSR representation.

However, these guidelines are also subject to a truncation for the storage limits of the other reservoir, as is briefly mentioned above. For this truncation we are trying to find the storage levels for our reservoir. These storage levels correspond to the MWVs just either side of where the truncation for the other reservoir's storage intersects the other reservoir's storage surface. This truncation is why we are representing points just either side of the truncation in our DSR and DSS representations rather than the truncation point itself. If the critical point just within the other reservoir's storage space is not stored then there is likely to be a loss of information near that reservoir bound. This is because where the other reservoir's storage limits is the constraining factor, the storage level shown in our reservoir's DSS remains constant from that storage point onwards.

These additional truncation points are also points at which significant changes occur in the DSS. Consequently the DSS must be reformulated for each guideline in terms of the DSR critical points, and the four sets of two critical points representing all four truncation boundaries on the DSS guideline. The DSR must be reformulated to likewise include these four sets of two critical points so that a cornerwise addition can occur which combines the DSS and DSR for the period to form the beginning of period EDSS for that reservoir. An identical set of operations must occur for the other reservoir's guidelines likewise. Again these truncation points will not unduly expand the total quantity of critical points that are preserved as the impact of these points will be softened by the inter-period stochastic inflows and another truncation will occur. This process will then result in points being excluded in the representation of the DSS as they become of decreasing importance when reforming the DSS for the previous period.

The SCDDP algorithm for the two reservoir problem becomes:

$Where\ t = Time, v_1 = Critical\ marginal\ water\ value\ res.1$
$v_2 = Critical\ marginal\ water\ res.2, r = res, r_1 = res$
$ev(s) = expected\ marginal\ water\ value\ as\ a\ function\ of\ storage$
$ev = critical\ expected\ marginal\ water\ value$

$$DSS'_{t,r,v_1,v_2} = DSS_{t,r,v_1,v_2} + DSR_{t,r,v_1,v_2}$$

$$DSS_{t-1,r,v_1,v_2,A} = \left(DSS'_{t,r,v_1,v_2} - inflow_{t,r,A}\right) \quad \forall t, \forall v_1, \forall v_2 \forall A$$

$$EMWV = ev(s)$$

$$EMWV = \sum_{A \in Inflow\ Set} Probability_A * MWV_s$$

$$\forall\ s \in Set\ of\ inflow\ adjusted\ Storage\ quantities\ associated\ with\ some\ v$$

$$EDSS_{t-1,r,ev_1,ev_2} = s(ev) \qquad \forall\ ev \in EMWV$$

$$if\left(EDSS_{t-1,r,ev_1e,v_2}\right) < Smin_r$$

$$truncate\ EDSS_{t-1,r,ev_1e,v_2} \forall ev_1\ to\ reflect\ Smin_r$$

$$if\left(EDSS_{t-1,r,ev_1e,v_2}\right) > Smax_r$$

$$truncate\ EDSS_{t-1,r,ev_1e,v_2} \forall v_1\ to\ reflect\ Smax_r$$

Algorithm 11 SCDDP for two reservoir problem

15. The Impact of Inter-Island Links On Two Reservoirs in Parallel (Double Filled LDC Extension)

The double filled LDC for two reservoirs where both reservoirs are located in one island is in most respects similar to the double filled LDC for a single reservoir. The second reservoir is yet again simply another option for generation in filling the LDC. The base scenarios of the critical prices derived from different positions with respect to thermals for this reservoir are established in the double filled LDC for single reservoir section above. However, alongside the prospect of this reservoir displacing thermals in the double filled LDC, there is also the prospect of the reservoir displacing the other reservoir. The complications inherent in a generic two-reservoir displacement and the corresponding cornerwise representation are discussed in depth in the two-reservoir CDDP discussion above.

The only real distinction between two reservoirs facing a single LDC and two reservoirs in the same area facing a double filled LDC is that a more complex series of tradeoffs occurs between the two reservoir releases when both reservoirs have an equal MWV. This reflects the formation of the DSR for the single reservoir facing a double filled LDC above. This trade off is shown in the LDC fill itself as is shown below. Were Hydro 1 to be displaced by Hydro 2 in the merit order of filling the local LDC then this displacement would occur where the MWV of Hydro 1 is equal to the MWV of Hydro 2. This is the marginal value for Hydro 1 and 2 generation as it is perceived where this generation fills load in the local LDC. However, a portion of Hydro 2's current position in filling the LDC means that the loss adjusted MWV of Hydro 2 is being traded off against the nominal SRMC of distant thermals. Given the position of Hydro 2 in the LDC we know that the release solution from Hydro 2 would only change with respect to local thermals at a MWV of 9 or a MWV of 10. However, the release solution could also change in situ were the loss adjusted MWV of Hydro 2 to increase such that it was greater than a distant thermal's SRMC or decrease such that it was less than a distant thermal's SRMC. Were Hydro 1 to displace Hydro 2 at an equal MWV then any load requirements with respect to distant thermals would be a component of the total load which would need to be filled by the combined releases of the reservoirs at that MWV.

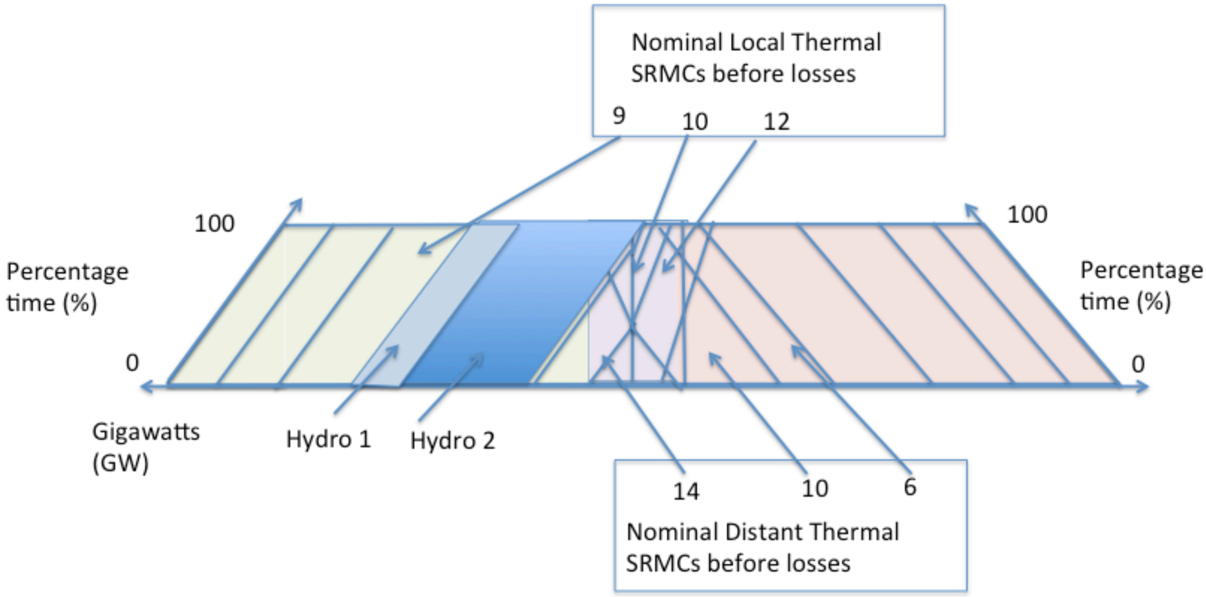


Figure 70: Tradeoff between Hydro 1 and Hydro 2 in filing the LDC

Yet again the critical thermal costs faced are the local values for local thermals, the loss adjusted values of exporting energy to the distant area and the loss-adjusted values of importing energy from the distant area. The distant areas thermals are backed off from supplying local load at the loss adjusted cost of importing energy; this is the MWV at which local hydro may displace distant thermals supplying local load. The distant thermals can also be backed off from supplying distant load at the loss adjusted cost of exporting energy; this is the MWV at which the local hydro displaces the distant thermals in supplying distant load. The local values are the MWVs at which local hydro displaces local thermals in the merit order for filling the LDC.

At each of these cost levels there can occur a trade off between the two hydro-electric reservoirs alongside those reservoirs' tradeoffs with the thermals. Thus for each reservoir there is a maximum and minimum release at that MWV if the other reservoir is positioned higher in the merit order of filling the LDC, and a maximum and minimum release at that MWV if the other reservoir is positioned lower in the merit order of filling the LDC. This is equivalent to the levels in using two reservoirs to fill a standard LDC apart from the transfer requirements outlined above.

As described in the above section on two reservoir CDDP, these maximum and minimum levels remain constant irrespective of how far above or below the other reservoir is in the merit order of filling the LDC. This mirrors the concepts established for generic single two reservoir LDC fill.

Notably, this DSR formation remains equally valid for the upstream-downstream two reservoir case discussed in greater depth below. This is because the actual trade off is constant irrespective of the interconnection of the two reservoirs. The upstream releases impact on the downstream reservoir level and so do impact on the relative positioning of both reservoirs within the merit order of filling the LDC. However, this formation of the DSR records the release level for both instances and correspondingly this impact is not taken into account in the pre-computed DSRs.

16. New Zealand Based Dual Parallel Reservoir Applications

The two reservoir system Matlab implementation of the CDDP and SCDDP algorithm was first verified to ensure that the results aligned with the expected results for a simple case with dummy data. This implementation was then applied to two different New Zealand two-reservoir system representations.

The verification of the model for CDDP involved running the system with a set of data with identical characteristics to that in R. A. Read, Dye, and Read (2012). The results from the CDDP calculation were identical to those produced by the model that was the basis for that paper. This verified that for a known set of deterministic system characteristics the model was producing the expected result. In particular it was noted that the truncation of the DSS occurred in line with the truncation occurring in R. A. Read, Dye, and Read (2012). .

In order to verify the model for SCDDP it was necessary to use a very simple stochastic representation of inflows. The system characteristics were limited such that the storage bounds would apply in the final time period. From this a manual composition of the EDSS values was created using excel. This was then used to verify that the EDSS formed in the final time period was equivalent to the result developed from a manual application of the algorithm. The simplification and addition were also evaluated based on a manual calculation of the expected value for these components. These then formed the EDSS' that would be passed back to another period. This verified that the intra and inter period problems were resolving in line with our expectations. However, as described above there was insufficient opportunity to more thoroughly investigate the loss of information resulting from the approximate representation of the EDSS'.

The New Zealand applications of the two reservoir system in which the two reservoirs are in the same island both included usage of the double reservoir LDC fill as describe above. The only distinctions are that a second reservoir is examined in terms of the optimal usage of stored and inflowing water. This means that a second generation source is excluded from the south island generation merit order. The sources used were in the first instance the Waitaki scheme and the Manapouri reservoir. There was also consideration of the Waitaki scheme and the Clutha Scheme.

For each MWV level faced by a reservoir in filling the LDC there is a maximum and minimum release level assuming that the other reservoir modeled is below this reservoir in the merit order of filling the LDC. Likewise there is a maximum and minimum release level assuming that the other reservoir modeled is above this reservoir in the merit order of filling the LDC.

The Waitaki scheme was modeled as described above, the Manapouri scheme reservoir characteristics and Clutha scheme reservoir characteristics were likewise based on the OPUS(2010) report.

Unfortunately the graphical components produced by the data as described above were unable to be graphically represented in a manner where guidelines were sufficiently differentiable so as to show the shape of the surfaces.

The computational time for a fairly complex cornerwise representation was under ten minutes. This time is reasonable however considerably exceeds that of earlier implementations of SCDDP.

17. Extension of Two Reservoir Constructive Dual Dynamic Programming and Stochastic Dual Dynamic Programming to Inter-Reservoir Transfer over an Inter-Island Link

The extension of the cornerwise CDDP algorithm to incorporate two reservoirs, each in a separate island with a constrained link between the two islands is entirely contained in the pre-computation phase of the DSRs. Fundamentally the structure of the CDDP algorithm itself remains constant: each reservoir faces a number of critical MWVs at which it can displace a generator in the merit order of filling the LDC, the value of storage is the future value that can be obtained by storing that unit of water, known inflows shift that storage surface and the storage surface must be truncated for reservoir limits. In a two reservoir system then the other hydro-reservoir is one of the generators that can be displaced in the merit order, and so for each reservoir maximum and minimum release and storage levels must be recorded for when the other reservoir is lower in the merit order, and higher in the merit order. All of this is identical in principle to the simple two reservoir case described above however the extent to which load can be filled by a reservoir in the other island, and the critical price levels at which the merit-order swap between the two reservoirs will occur will be affected by the capacity constraints and losses on the HVDC link. These considerations will be taken into account in the DSR pre-computation as described below.

The extension of SCDDP to incorporate two reservoirs is likewise entirely contained within the formation of the DSR. The only distinctions between this algorithm and the CDDP algorithm extension are those outlined in the SCDDP section for two reservoir SCDDP above.

It is clear that the reservoir configuration and characteristics are internalized in the DSR formation phase. Once this is formed from the double filled LDC then the CDDP and SCDDP algorithms as described above can readily be applied to produce an appropriate DSS.

18. The Impact of Inter-Reservoir Transfer over an Inter-Island Link on Filling the LDC

In implementing a double filled LDC for two reservoirs each located in a different island there are a number of factors that increase the complexity of the representation and the interpretation thereof. There is no longer a one-to-one ratio where one reservoir displaces the other in the merit order, a reservoir can only displace a constrained portion of the other reservoir's generation and the impact of different reservoir combinations on link utilization is less intuitive.

To explore the implications of two hydro-electric reservoirs each in a different island on the filling of a double LDC we first assume there exists a unidirectional link between the islands. The hydro reservoir in one island will be described as the distant reservoir, this reservoir can only fill load which is formed from the LDC in its own island. The other reservoir will be referred to as the local reservoir. The local reservoir faces loads at two different efficiencies. This reflects that the load in the distant region is adjusted to account for losses, while the local load is not. We note however, that this is very similar to the scenario of an LDC that would be faced if the reservoir releases could produce electricity at two different efficiency levels.
Where the distant reservoir cannot transfer generation in to the local region then the distant reservoir solution will change in only two circumstances. Either the MWV of the reservoir is equal to the marginal cost of a thermal in the distant region, or the loss adjusted critical value of a thermal or hydro-generation from the local region displaces the distant reservoir in filling the distant LDC. The latter displacement would occur where the loss-adjusted SRMC or MWV of the local generation source was equal to or less than the distant reservoir's MWV. This means that from the perspective of the distant reservoir the filling of the distant LDC is occurring in an identical fashion to how the filling would occur in the generic two reservoir CDDP formulation. As far as that reservoir is aware, there is no other island, and the generation sources coming from that island are simply more thermal generation sources that happen to only be able to generate collectively up until the capacity

constraint on the link. This is shown diagrammatically below, firstly from the perspective of filling the distant LDC alone, and then in terms of the double filled LDC. The inter-island transfer is incapable of filling the full time requirement for that load, however it can fill a portion of it. Consequently where this load replaces a component of a generator's load in the merit order fill that causes a reduction in the % of time that generator's facility is required for. Effectively, where available at a lower SRMC, the transferred energy is preferred. Where the transferred energy is not available at the lower SRMC then the load is filled in the standard local merit order.

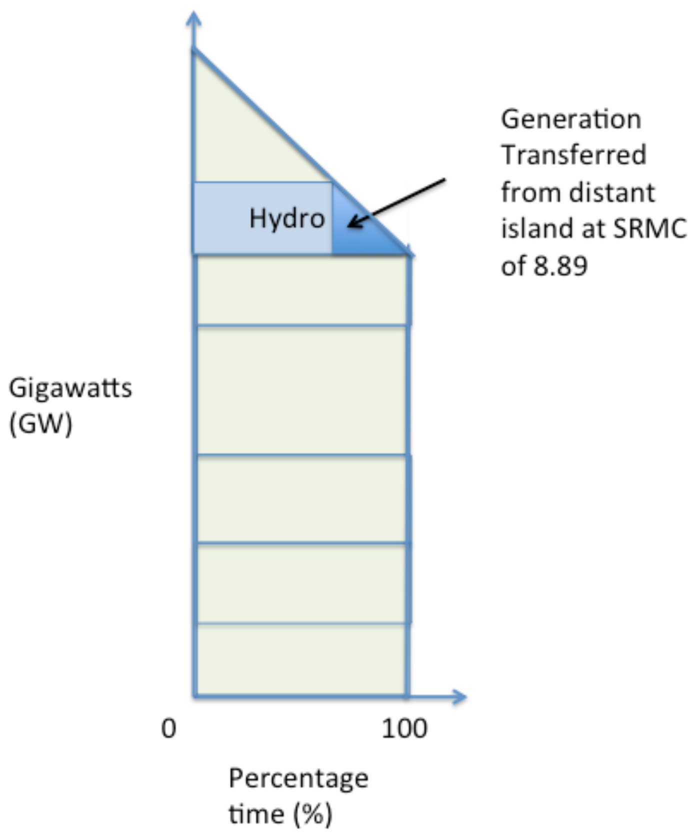


Figure 71: Interisland transfer filling load, single LDC

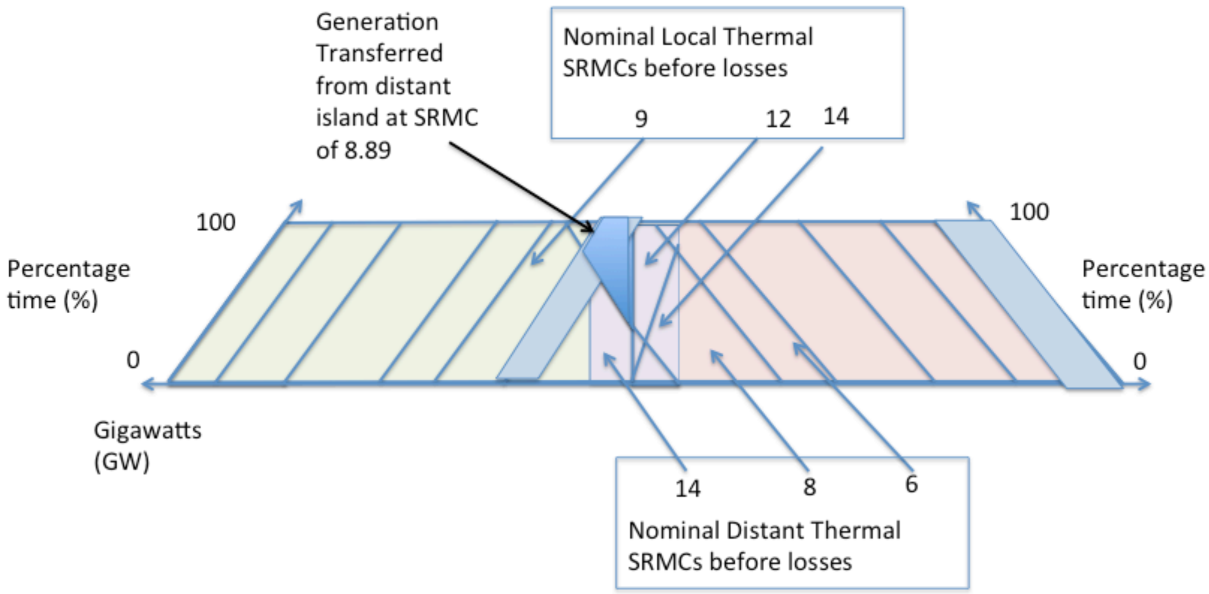


Figure 72: Inter-island transfer displacing local load.

In contrast the where the local reservoir is filling load in the local LDC then a larger number of possible areas for solution change exist. The local reservoir can displace thermal generation in the local LDC at the SRMC of that generation source. The local reservoir can also displace thermal generation transferred from the distant LDC at the loss-adjusted SRMC of that generation source. The local reservoir can alternatively displace thermal generation filling the distant LDC where the loss-adjusted MWV of the reservoir is equal to the SRMC of that generation source. All of these alternatives are equivalent to using a double filled LDC for two reservoirs in parallel as described above. However, there is also the prospect of displacing the distant reservoir in filling the LDC. This too would occur where the loss-adjusted MWV of the local reservoir was equal to the MWV of the distant reservoir.

The key price relationships are indicated when the equivalent trade off is occurring in both reservoir spaces.

Action	Local Reservoir	Distant Reservoir
Displacing Local Generation filling Local load	Critical SRMCs of Local generation	MWVs at which loss-adjusted MWV of reservoir is equal to critical SRMCs of Local generation

Displacing Local Generation filling Distant load	Critical SRMCs of Local generation	Loss Adjusted critical SRMCs of Local generation
Displacing Distant Generation filling Distant load	MWVs at which loss-adjusted MWV of reservoir is equal to critical SRMCs of Distant generation	Critical SRMCs of Distant generation
Displacing Distant Generation filling Local load	Loss Adjusted Critical SRMCs of Distant generation	Critical SRMCs of Distant generation

Table 4 Potential displacement actions and the corresponding MWVs for each reservoir

The MWVs faced by the local reservoir for displacing load that is being filled in the distant island by local generation are the same as those at which local generation is being displaced. This is because both generation sources go through the same loss adjustment processes. Consequently if a local generator is generating 100 GWh at $10 locally to fill the load in the distant island, then the hydro-electric reservoir will have to generate an equivalent amount at an equivalent price in order to displace that generator in the merit order. However, the resultant critical price as perceived from the other island will be the loss-adjusted price for either of these generation sources. Likewise the resultant quantity that would arrive would also be adjusted for losses.

These differing critical price levels and the relationship between them must be taken into account when constructing the DSRs for each reservoir respectively. By taking this into account in the DSR stage the computational burden of considering a two reservoir two area model is taken into account in a single pre-computation rather than it being necessary to take it into account at every stage. This means that there is little distinction in efficiency between a simpler and more complex reservoir configuration which models a constrained link with losses.

18.1 Forming DSRs with a unidirectional link

There is a different DSR formation process for each reservoir where they exist in separate islands with a unidirectional loss-heavy link joining the two islands. The process for forming the DSR for a reservoir which can supply through a loss adjusted link differs from the process for forming the DSR for the reservoir faced with supply through a loss adjusted link. For the purposes of this discussion, we will assume that Hydro 1 is the local reservoir transferring through such a link and Hydro 2 is the distant reservoir.

For the distant reservoir, where a reservoir is only subject to electricity supplied through a loss adjusted link, two sets of critical MWVs are relevant. These are comparable to those described above for a simple one reservoir case. For one of these DCRs, the critical MWVs will be the marginal costs of the thermals in the distant region, and the quantities would be the marginal reservoir release quantities. These quantities correspond to the base quantity that can be released at the given MWV for that reservoir, to generate the equivalent quantity of that generator which is being displaced.

The second set of critical marginal values for the distant reservoir will be the loss-adjusted critical SRMCs or MWVs of generators in the local area filling the distant load. The actual critical marginal cost value faced will be the value at which the MWV of the distant reservoir is equal to the SRMC or MWV in the local island adjusted for losses. This is the MWV at which there is a point of equivalence between filling the load using the distant source or releasing the reservoir's generated capacity. The quantities that will be supplied by the reservoir if the reservoir displaces the local generation in the merit order is equal to the loss adjusted generation quantities from the local generator. This is the quantity of distant load that was actually being supplied.

These two critical marginal value lists and their associated marginal release quantities are then horizontally concatenated and all columns are sorted according to the MWV level, from lowest to highest value. The marginal release quantities at each value are cumulated to form the total release at each MWV. In the cumulating procedure the lowest total release will be the release associated with the highest MWV and vice versa. Thus for the single reservoir, a DCR for filling local load when generation can be supplied to fill that load from another island at a known loss level can be formed.

Note that the inter-island link in practice constrains the transfer between islands. This means that the local island can only supply that constrained amount before loss-adjustments are applied while the rest of the load is supplied by distant generators. The assumption is that it is this total quantity produced including losses that must be truncated to account for the constraints of the inter-island link. This is so that the line will not be overloaded at any point.

For the local reservoir there are likewise two sets of critical cost levels at which the release solution for this reservoir will change. The first of these is the set of critical SRMCs associated with the local generation sources. The quantities associated with these are the additional quantities that would be generated by these generation sources had they not been displaced in the merit order. In filling the local load this is equal to the quantity of load that has been fulfilled. However, in displacing a local thermal generator that is filling load in the distant island, the quantity is equal to the sum of the load that is being filled in the distant island and any losses incurred during transfer.

The second set of critical cost levels at which the release solution for this reservoir will change is where the loss-adjusted MWV of the local reservoir is equal to the SRMC or MWV of the distant generator that it has displaced in the merit order. These are the critical costs faced in filling load in the distant island that was initially being supplied by a generator in the distant island. The marginal quantity that would be associated with that displacement is the quantity that must be generated by the local reservoir in order to fulfil the load that would have been fulfilled by the displaced generator. Note that in practice this quantity is constrained by the inter-island link. The quantities that are supplied associated with these MWVs are lower than the quantity generated because of the impact of losses. Consequently the quantity that is generated must include both the quantity supplied and the quantity that will be lost in transfer. In practice the quantities associated with this second set of marginal values, if stored in terms of the quantity of energy produced, must be scaled by the loss level ratio. This ensures that all water release quantities are in comparable units: number of units of energy produced, as opposed to number of units of energy supplied.

As for the distant reservoir two critical marginal value lists and their associated marginal release quantities are then horizontally concatenated and all columns are sorted according to the MWV level, from lowest to highest value. The marginal release quantities at each value are cumulated to form the total release at each MWV. In the cumulating procedure the lowest total release will be the release associated with the highest MWV and vice versa. Thus for the single reservoir, a DCR for filling local load when generation can be supplied to fill that load from another island at a known loss level can be formed.

However, in practice the local island can only supply that constrained amount before loss-adjustments are applied while the rest of the load is supplied locally. As mentioned above the assumption is that it is this total quantity produced including losses that must be truncated to account for the constraints of the inter-island link. This is so that the line will not be overloaded at any point.

The impact of adding hydro capacity in the local island is lowering the cost of the over-all solution with the additional possibility of increasing the utilization of the inter-island link. This reflects that the local generation sources overall have become cheaper than they were relative to the distant generation. Conversely the impact of adding hydro capacity in the distant island is lowering cost of the over-all solution with the additional possibility of decreasing the utilization of the inter-island link. This reflects that the addition of hydro-electric generation to the distant island has meant that distant generation on the whole has become cheaper than it was relative to the local generation.

18.2 Forming DSRs with a Bi-Directional Link

Forming DSRs with a bi-directional link can be naturally developed by looking at each side of the two directional case. Where the reservoir can have energy supplied by the other island then it must face the critical SRMCs of generation in its own island, and the loss-adjusted SRMCs of generation that can be supplied to that island from the other island. The marginal quantities associated with these are respectively the quantity of additional load which the displaced generator was fulfilling prior to its displacement, once inter-island link constraints are taken into account. Where the reservoir can supply energy to the other island then it must face the critical SRMCs of generation in its own island, and the other critical values are where the loss-adjusted reservoir MWV is equal to the critical SRMCs of generation sources in the other island. The marginal quantities associated with these are respectively the quantity of additional load which the displaced generator was fulfilling prior to its displacement, and the lesser of, the sum of that quantity and any losses incurred in the interisland link, and the quantity constraint on that link. Consequently, for a reservoir which can both supply energy, and have energy supplied all three of these must be incorporated.

The means that for the local reservoir the critical MWVs are the following:

- The critical SRMCs of local generation
- The loss-adjusted SRMCs of generation that can be supplied to the local island from the distant island
- The values at which the loss-adjusted MWV of the local reservoir is equal to the SRMC of a source of generation in the distant island

The corresponding marginal quantity levels are:

- The quantity of local load being supplied by the local generation source
- The quantity of local load being supplied by the distant generation source
- The lesser of:
  - The sum of the quantity of the distant load being supplied by the distant generation source and the losses incurred in transfer over the link

- The capacity quantity constraint on the inter-island link multiplied by the percentage of the time period the distant load is filled

As in the above cases the critical marginal value lists and their associated marginal release quantities are then horizontally concatenated and all columns are sorted according to the MWV level, from lowest to highest value. The marginal release capacity at each value is cumulated to form the total release at each MWV, until the total quantity associated with the third MWV category established above is equal to the quantity constraint on the inter-island link. In the cumulating procedure the lowest total release will be the release associated with the highest MWV and vice versa. Thus for the single reservoir, a DCR for filling local load when generation can be supplied to fill that load from another island at a known loss level can be formed. Unlike where both reservoirs are in parallel, the outcomes for the inter-island link are less evident for the two reservoir problem with a reservoir in each island. Even if a hydro reservoir is also providing generation capacity in the other area, and has a sufficiently low MWV to be included in the LDC fill, the displacement may not impact the quantity of energy transferred between the areas. When displacement does occur, the displacement in one island caused by the inclusion of a reservoir may be largely offset by the displacement caused in the other by the inclusion of a reservoir there. Consequently, the primary impact that the inclusion of hydro in both areas has on the electricity system is that the cost of filling the LDC may be lowered.

19. New Zealand Based Dual Reservoir Application with Inter-Reservoir Transfer over an Inter-Island Link

In the two area two reservoir application of the SCDDP and CDDP algorithms to the New Zealand system, there were no major algorithmic changes. Consequently it was assumed that the verification of the algorithm performed for the earlier two reservoir case was sufficient to cover the two area two reservoir MatLab implementation likewise. The major difference between the two area two reservoir implementation and the two reservoir implementation more generally is in the formation of the DSRs from the double-filled LDC.

This means that the DSRs are formed as described in the doubled-filled LDC section above for a reservoir in the North Island and a reservoir in the South Island. In this case we have implemented this with the Waitaki scheme as the South Island reservoir using the Waitaki scheme characteristics described above. The North Island reservoir used is an aggregation of all major North Island Reservoirs mentioned in OPUS (2010). The HVDC link is assumed to have a capacity of 1000 MW.

Unfortunately the graphical components produced by the data as described above were unable to be graphically represented in a manner where guidelines were sufficiently differentiable so as to show the shape of the surfaces. However, in earlier simpler cornerwise representations it was noted that the storage guideline for each reservoir primarily reflected the filling of local demand as only a small portion of the storage capacity could be used for transfer.

The computational time for a fairly complex cornerwise representation was under ten minutes. This time is reasonable however considerably exceeds that of earlier implementations of SCDDP (around 2 minutes). Nonetheless it is far lower than alternative solutions for the reservoir management problem that typically take hours or days to solve.

20. Constructive Dual Dynamic Programming for Two Reservoirs in Series

The extension of the two reservoir implementation of CDDP to incorporate the simple case where both reservoirs are on a single river system is relatively intuitive. The assumptions surrounding our implementation of the model are that all the outflows from the upper reservoir arrive in the lower reservoir, and that they arrive within the time period in which they are released.

Where all flows arrive within the period within the time period that they are released then the DSR of the upstream reservoir can be included as a known inflow to the downstream reservoir. This means that while the CDDP algorithm would remain the same for the upstream reservoir, the CDDP algorithm for the downstream reservoir would be as is reflected in Algorithm 12 below.

$$Where\ t = Time, E = Efficiency\ ratio$$
$$u = upstream\ reservoir, d = downstream\ reservoir$$

$$DSS'_{t,u,v_1,v_2} = DSS_{t,u} + DSR_{t,u}$$
$$DSS_{t-1,u,v_1,v_2} = \left(DSS'_{t,u,v_1,v_2} - inflow_{t,u}\right) \quad \forall v_1, \forall v_2$$
$$if\ \left(DSS'_{t,u,v_1,v_2} - inflow_{t,u}\right) < Smin_u$$
$$truncate\ DSS_{t-1,u,v_1,v_2} \forall v_1\ to\ reflect\ Smin_u$$
$$if\ \left(DSS'_{t,u,v_1,v_2} - inflow_{t,u}\right) > Smax_u$$
$$truncate\ DSS_{t-1,u,v_1,v_2} \forall v_1\ to\ reflect\ Smax_u$$

$$DSS'_{t,d,v_1,v_2} = DSS_{t,d,v_1,v_2} + DSR_{t,d,v_1,v_2}$$
$$DSS_{t-1,d,v_1,v_2} = (DSS''_{t,d,v_1,v_2} - inflow_{t,d} - DSR_{t,u,v_1,v_2}E)$$
$$if\ (DSS''_{t,d,v_1,v_2} - inflow_{t,d} - DSR_{t,u,v_1,v_2}E) < Smin_d$$
$$truncate\ DSS_{t-1,d,v_1,v_2} \forall v_2\ to\ reflect\ Smin_d$$
$$if\ (DSS'_{t,d,v_1,v_2} - inflow_{t,d} - DSR_{t,u,v_1,v_2}E) > Smax_d$$
$$truncate\ DSS_{t-1,d,v_1,v_2} \forall v_2\ to\ reflect\ Smax_d$$

Algorithm 12 Upstream Downstream DSS formation alteration

Until this stage we have considered units of water in equivalent energy terms. However, where reservoirs are in series the same quantity of water released by the upstream reservoir will arrive in the downstream reservoir. If the generation facilities upstream from the downstream reservoir do not operate with the same efficiency as those downstream from the downstream reservoir then the energy equivalent quantity will need to be scaled so as to reflect the real energy that can be produced by that unit of water in the downstream reservoir. Thus upstream release quantities must be scaled by an efficiency ratio, E, when the additional inflow is taken into account on receipt in the downstream reservoir.

For clarity it is also worth noting that the unit of water in the upstream reservoir contains both the generation potential for the upstream generation and for the downstream generation. This potential is represented by including both these components in the upstream MWV. Hence the critical ratio between the two MWV’s is where the difference between the upstream and downstream MWV’s is equal to the downstream MWV.

The arrival of these known inflows implies a 'diagonal' shift on the storage diagram. This is due to a reduction in upstream storage levels and corresponding increases in downstream storage levels. That same diagonal shift must be reflected in the way the DSSs are interpreted in the CDDP algorithm.

The release surfaces are largely identical in form to those of two-reservoirs in parallel described above. At a fundamental level the benefit and generation from these reservoir releases is treated in an identical fashion in the market to any other generation, and hence the double filled LDC filling process is in most respects identical. The release from the downstream reservoir is still driven by its MWV, and so this is identical to the parallel LDC fill process. However, release from the upstream reservoir is now driven by the difference between its MWV and that downstream as described above.

This result in the guidelines for storage in the upstream reservoir corresponding to points at which the efficiency adjusted MWV difference between upstream and downstream storage is equal to a critical level. If the upstream and downstream

MWV are equal, this now means the additional value of holding water upstream is zero, implying that we are indifferent as to whether or not upstream water spills. However the critical price relationship, at which we now are indifferent between filling the same load from upstream or downstream generation sources is now when the downstream MWV equals the (efficiency adjusted) difference between upstream and downstream MWVs. At that storage balance point the MWV ratio is (1+E), implying that energy stored in the upstream reservoir has just the same marginal value as energy stored in the downstream reservoir. The core, or shaded area in Figure 62 below represents the levels at which the marginal value of releasing water downstream is equal to the marginal value of releasing water upstream. This is where the switch in the merit order between the two reservoirs would occur.

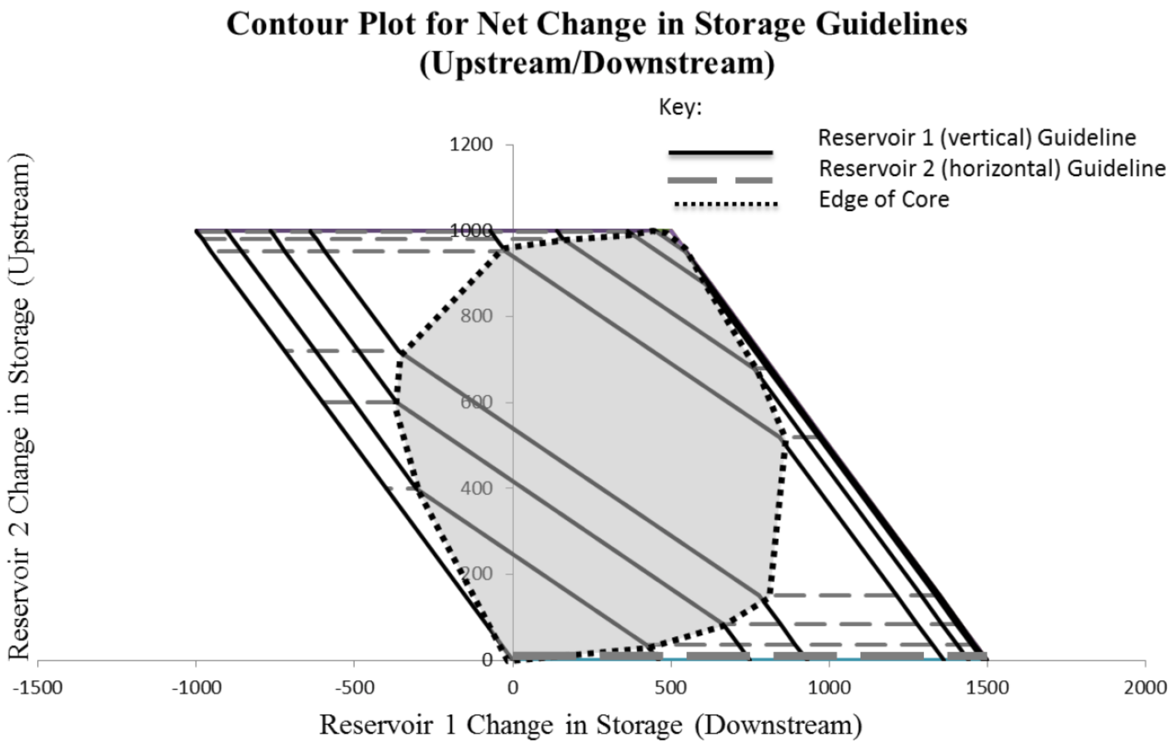


Figure 73 Deterministic upstream downstream storage guideline diagram, originally published (R. A. Read et al., 2012)

The upstream releases also impact downstream storage differently. The change is displayed in the storage diagram above: we get the sheared version of the general storage diagram, where Hydro 2 is the upstream reservoir and E= 1.Where upstream release is greater than downstream release the net result of releases for the downstream reservoir can be an increase in the storage level.

The guidelines for the upstream reservoir remain horizontal, but now correspond to a constant MWV difference in the upstream reservoir. However the shape of the guidelines corresponding to constant MWV in the downstream reservoir is no longer constant where no merit-order swap occurs. Instead they have a slope reflecting the efficiency ratio between the two reservoirs. The area of indifference between upstream and downstream release at the same MWV still occurs. However the shapes of these have changed, they have a shallower slope. Before downstream storage could be raised indirectly by backing off downstream release and supplementing it with upstream release. However in the upstream-downstream configuration then such a measure would result in the release of more upstream water, which would in turn be an inflow to the downstream reservoir raising the storage level of that reservoir directly. With that understanding, the upstream-downstream algorithm given above requires a slight clarification.

The storage bounds to which guidelines must be truncated are not sheared. So the 'constant' aspects of downstream guidelines (where no merit order swap is occurring) must now be truncated to the bounds based on the reservoir storage limit and the projection from that limit for the other reservoir at a known angle. This is similar to the truncation of the guideline component at which there is a trade off between the two reservoir capacities.

21. Stochastic Constructive Dual Dynamic Programming for Two Reservoirs in Series

The extension from the SCDDP algorithm for two reservoirs in parallel to the SCDDP algorithm for two reservoirs on the same river system is largely identical to the equivalent extension for CDDP described above. This is shown mathematically and below:

$E = Efficiency\ ratio$
$u = upstream\ reservoir, d = downstream\ reservoir$
$ev(s) = expected\ marginal\ water\ value\ as\ a\ function\ of\ storage$

$ev = critical\ expected\ marginal\ water\ value$

$$DSS'_{t,u,v_1,v_2} = DSS_{t,u,v_1,v_2} + DSR_{t,u,v_1,v_2}$$

$$DSS_{t-1,u,v_1,v_2,A} = \left(DSS'_{t,u,v_1,v_2} - inflow_{t,u,A}\right) \quad \forall t, \forall v_1, \forall v_2 \forall A$$

$$EMWV = ev(s)$$

$$EMWV \ = \sum_{A \in Inflow\ Set} Probability_A * MWV_s$$

$$\forall\ s\ \in Set\ of\ inflow\ adjusted\ Storage\ quantities\ associated\ with\ some\ v$$

$$EDSS_{t-1,u,ev_1,ev_2} = s(ev) \qquad \forall\ ev\ \in EMWV$$

$$if\left(EDSS_{t-1,u,ev_1 e,v_2}\right) < Smin_u$$

$$truncate\ EDSS_{t-1,u,ev_1 e,v_2} \forall ev_1\ to\ reflect\ Smin_u$$

$$if\left(EDSS_{t-1,u,ev_1 e,v_2}\right) > Smax_u$$

$$truncate\ EDSS_{t-1,u,ev_1 e,v_2} \forall v_1\ to\ reflect\ Smax_u$$

$$DSS'_{t,d,v_1,v_2} = DSS_{t,d,v_1,v_2} + DSR_{t,d,v_1,v_2}$$

$$DSS_{t-1,d,v_1,v_2,A} = \left(DSS'_{t,d,v_1,v_2} - inflow_{t,d,A} - DSR_{t,u,v_1,v_2}E\right)$$

$$\forall t, \forall v_1, \forall v_2 \forall A$$

$$EMWV = ev(s)$$

$$EMWV \ = \sum_{A \in Inflow\ Set} Probability_A * MWV_s$$

$$\forall\ s\ \in Set\ of\ inflow\ adjusted\ Storage\ quantities\ associated\ with\ some\ v$$

$$EDSS_{t-1,d,ev_1,ev_2} = s(ev) \qquad \forall\ ev\ \in EMWV$$

$$if\left(EDSS_{t-1,d,ev_1e,v_2}\right) < Smin_d$$

$$truncate\ EDSS_{t-1,d,ev_1e,v_2} \forall ev_2\ to\ reflect\ Smin_d$$

$$if\left(EDSS_{t-1,d,ev_1e,v_2}\right) > Smax_d$$

$$truncate\ EDSS_{t-1,d,ev_1e,v_2} \forall v_2\ to\ reflect\ Smax_d$$

Algorithm 14 Upstream Downstream SCDDP Formulation

Likewise as described above there will be no alteration to the shape of the DSR for these reservoirs. However, the upstream critical price levels will now be considered in terms of the difference between the upstream MWV and downstream MWV as the trade off critical points. The merit order swap between the upstream and downstream reservoir will occur when the upstream MWV is twice the downstream MWV assuming that E=1.

In formulating the EDSS however, it is necessary that the reservoir outflow from the upstream reservoir is considered as an additional known inflow for the purposes of the inflow shift. There are a number of possible methods of integrating this inflow into the downstream reservoir. It is likely that where reservoirs in series are situated relatively proximate that there may be a strong correlation between the inflow for the two reservoirs and hence one possibility is that for each inflow scenario the downstream reservoir MWVS would be shifted by the corresponding upstream release amount and the corresponding inflow. This would then be associated with a probability weighted price from which the EDSS would be formed in accordance with the description for two reservoir SCDDP above. The other key method is assuming that there is no correlation between inflows and simply subtracting the release amount which is associated with the equivalent EMWV. The reality of the situation is likely to be somewhere between these two methods. However, to promote simplicity in our implementation of this algorithm we have elected to assume that inflows are not correlated within each period.

The complications involved in the truncation for reservoir limits are, as described above, largely overcome by the use of two critical points to represent each corner. This allows a reasonable approximation of the surfaces. The application of the

SCDDP method for reformulating the EDSS in order to allow later addition of the DSR is identical to that for the general two reservoir SCDDP methodologies described above. As the SCDDP guidelines are less clearly representative of explicit tradeoffs there is no need to adjust the truncation method described for SCDDP above

22. New Zealand Based Application for Two Reservoirs in Series

The MatLab implementation of the upstream-downstream 2 reservoir system the results were verified for a single CDDP case and the implementation was used to represent the upstream-downstream component of a simplified Waitaki system. The only aspect of the upstream-downstream case that distinguishes the upstream-downstream algorithm from the more generic algorithm is that the DSR from the upstream reservoir formed part of the inflows to the downstream reservoir. We verified that this calculation was performed correctly in the deterministic upstream-downstream case by comparison to the results that were the basis of R. A. Read, Dye, and Read (2012). All other aspects unique to the deterministic and stochastic algorithm implementation were identical to the verified results above where the upstream-downstream component was removed. Thus the generic upstream-downstream CDDP and SCDDP implementation is operating in the expected manner. The New Zealand application of the upstream-downstream algorithm used the Waitaki scheme divided into an upstream reservoir and a downstream reservoir in a simplistic manner. For this implementation we have assumed that the Lake Tekapo and Lake Pukaki have the storage capacity limits indicated in OPUS(2010). We further note that while in practice it is likely that the inflows to each of these lakes will be closely correlated, in order to preserve the simplicity of the model we have assumed that the inflows are independent. This is primarily a measure to ensure that the upstream-downstream model could be readily verified against previous implementations. In the interests of simplicity we ignore the possibility that upstream spill could by-pass downstream generation and assume that all upstream release arrives in the downstream reservoir, in the same period. Generation efficiency is assumed to be constant, for each reservoir. Although the same quantity of water may produce a different quantity of energy at each reservoir we assume that the water is stored in energy equivalent terms. Thus upstream release quantities are scaled by an “efficiency ratio”, E, on receipt downstream. In practice as a result of the nature of information available in OPUS (2010), E for the Waitaki system has a value of 1. All other key data is as described of the two reservoir cases

above.
Unfortunately the graphical components produced by the data as described above were unable to be graphically represented in a manner where guidelines were sufficiently differentiable so as to show the shape of the surfaces.
The computational time for a fairly complex cornerwise representation was under ten minutes. This time is reasonable however considerably exceeds that of earlier implementations of SCDDP.

23. Beyond Two Reservoirs

The extension of the CDDP and SCDDP concepts beyond the one and two reservoir implementations described here has primarily not been feasible due to limitations in visualizing the problem. The use of critical points is intended to capture the shape of the space without necessarily needing to know or visualize the intervening space. This can be seen in the diagram below for a 3 dimensional case. Provided that for every critical corner + - ε points are known in every direction then the surfaces can be recreated. This allows solutions to be developed and applied for higher reservoir models.

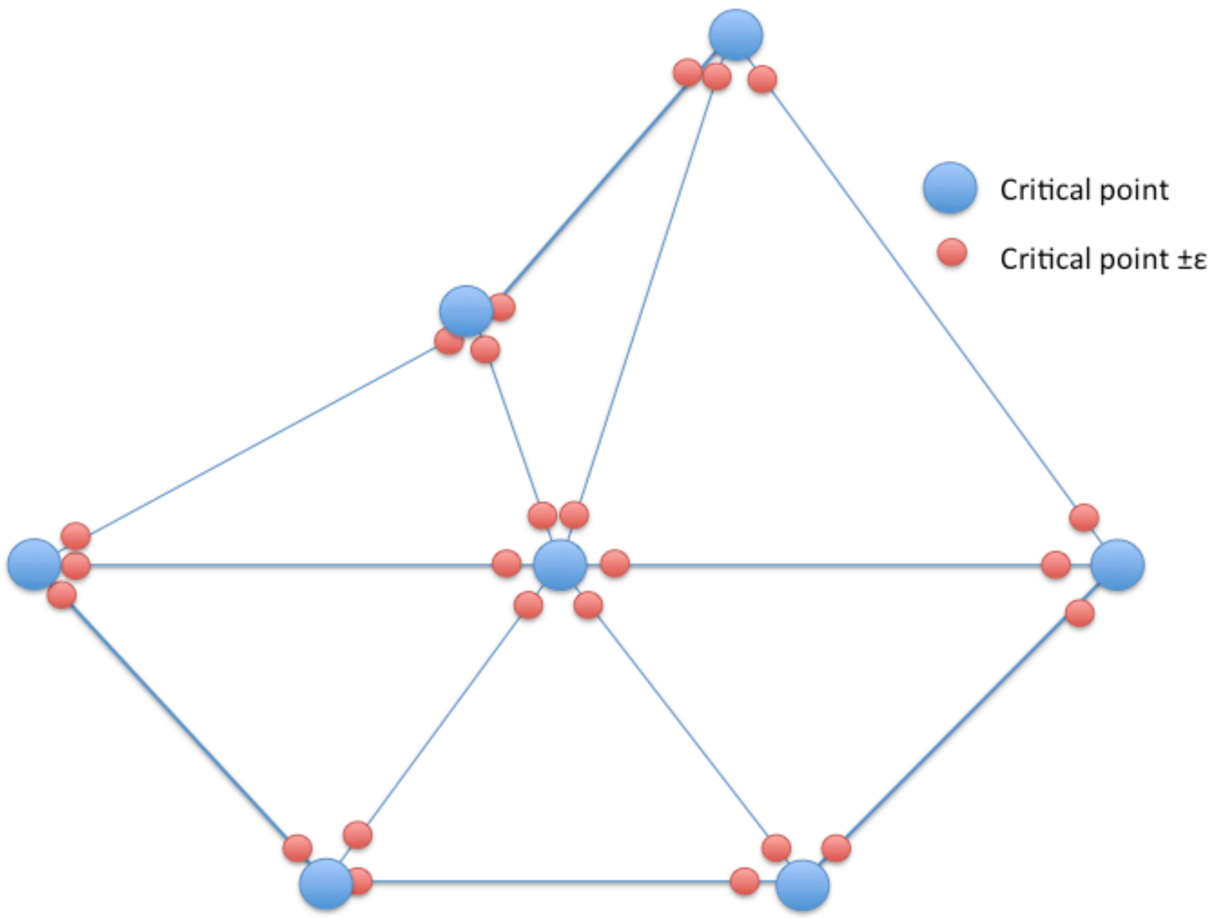


Figure 74 Critical points for three reservoir DSS representation

The primary identified issue in extending the CDDP algorithm beyond two reservoirs is that at present a coherent and well-considered method of truncation for these scenarios has not been developed. At present truncation occurs at a point for the reservoir for which the limit applies. The truncation also occurs over a number of possible values for the other reservoir. Where there are three reservoirs then when one reservoir is constrained by storage limits then it is constrained over a range of storage values for the other two reservoirs. The storage values of the other two reservoirs form a surface over which the storage value from the third reservoir is constant.

The underlying issue with this truncation is that considerable thought must be given to determine the minimum number of points that must be recorded to represent this truncation fully in the solution and what these points are. Consideration must likewise be given to how this point selection process would extend to higher dimensional spaces. Efficiently selecting these points would be the first step necessary to continue this research to incorporate higher reservoir numbers. There are likewise issues in extending the SCDDP algorithm as written beyond two reservoirs. These issues centre on the selection of the appropriate variations from the established critical points. The first of these is in selecting the value for E. This would have increasing significance with the extension to higher reservoirs as each reservoir may have a distinct inflow pattern that requires a distinct E value.

Correspondingly there would need to be further investigation into the implications of approximating the DSS in accordance with only the DSR critical points and the points to reflect truncation. In particular issues may arise where there are significant changes between these critical corners. Intuitively this is more likely to occur as the reservoir numbers increase. This increase would be due to the combined impact of the potential storage levels for different reservoirs in a single area.

Another interesting further area for study would be an exploration of the relative strengths and weaknesses of SDDP and SCDDP. At present there have been no comprehensive benchmarking exercises with the two models. This would particularly aid in realizing which model is more likely to give a useful result for different reservoir configurations and constraints. However, it should be noted that any attempt to compare the results would have to include a specified set of heuristics for the application of transmission constraints and possibly even higher reservoir numbers to the SCDDP solution.

24. Conclusions

Through the exploration of representing significant changes of in the DSR and DSS diagrams by a set of critical points that capture the change implied it should be feasible to construct SCDDP algorithms that allow for the extension of SCDDP to incorporate higher reservoir numbers. The development of alternative SCDDP reservoir configurations in the context of the New Zealand system also expands a number of the underpinning structures to the conceptualization of current SCDDP implementations. In particular it has motivated the development of the double filled LDC technique to ensure that thermal generation levels from both islands can be incorporated. This enables the further development of SCDDP as a potential planning tool to determine the value of establishing such generation facilities. Furthermore the exploration of generalizing the SCDDP algorithm to incorporate reservoir in series at furthers the conceptual framework. This may even enable the further development of SCDDP to larger river systems or at least inform the decision-making policies of operators of reservoirs in series. This could be particularly beneficial in New Zealand as a large proportion of New Zealand's hydro-electric generation potential is held in a number of reservoirs in series on the Waitaki river system.

Key areas identified for further research would be the extension of the SCDDP cornerwise algorithm to a three-reservoir configuration in parallel, an exploration of more effective modeling of the Waitaki river chain in a mid-term and long-term optimization model and the incorporation of stochastic correlation to the algorithm as described above. The incorporation of stochastic correlation is likely to have particular utility in considering the operational guidelines of reservoirs in series.

25. Bibliography

Bannister, C. H., & Kaye, R. J. (1991). A Rapid Method for Optimization of Linear systems with Storage. *Operations Research, 39*(2), 220-232.

Cosseboom, P. D., & Read, E. G. (1989). *Dual Dynamic Programming for Coal Stockpiling*. Paper presented at the ORSNZ Proceedings.

Craddock, M., Shaw, A., Telfar, G., Hartley, M., Pedder, S., & Winter, N. (1998). *RAGE 2 Project Report*. Strategic Development. Electricity Authority of New Zealand.

Culy, J., Willis, V., & Civil, M. (1990). *Electricity Modeling in ECNZ Re-visited*.

Culy, J. G., Read, E. G., Wright, B. D., & New Zealand Institute of Economic, R. (1995). *The evolution of New Zealand's electricity supply structure* (Vol. 94/33.). Wellington, N.Z , NZ Institute of Economic Research.

Dreyfus, S., & Law, A. (1977). The art and theory of dynamic programming. *Academic, New York*.

Dye, S., & Read, E. G. R., R.A. Starkey, S.R. (2012). *Easy Implementations of Generalised Stochastic CDDP Models for Market Simulation Studies*. Paper presented at the Paper presented at the 4th IEEE/Cigré International Workshop on Hydro Scheduling in Competitive Markets, Bergen, Norway.

Energy Data File. (2012). Retrieved 21/03/2013, from Ministry of Economic Development http://www.med.govt.nz/sectors-industries/energy/pdf-docs-library/energy-data-and-modelling/publications/energy-data-file/energydatafile-2011.pdf

Generating Station List as at September 2012. (2012). Retrieved 21/03/2013, from Electricity Authority of New Zealand http://www.ea.govt.nz/industry/monitoring/forecasting/long-term-generation-development/list-of-generation-projects/

Halliburton, T. (1994). Medium Term Generation Scheduling Options or Alternative to SPECTRA (F. R. Group, Trans.). Wellington, New Zealand: Electricity Corporation of New Zealand.

Huang, W., Harboe, R., & Bogardi, J. (1991). Testing stochastic dynamic programming models conditioned on observed or forecasted inflows. *J. Water Resour. Plan. Manage, 117*(1), 28-36.

Kerr, A. L., Read, E. G., & Kaye, R. J. (1988). *Reservoir Management with Risk Aversion*. Paper presented at the ORSNZ Proceedings.

Labadie, J. (1993a). Combining simulation and optimization in river basin management. *Stochastic hydrology and its uses in water resources systems simulation and optimization, J. Marco et al., eds., Kluwer Academic, Dordrecht, The Netherlands, 345–371.*

Labadie, J. W. (2004). Optimal Operation of Multireservoir Systems: State-of-the-Art Review. *Journal of Water Resources Planning and Management, 130*(2), 93-111. doi: 10.1061/(ASCE)0733-9496(2004)130:2(93)

Lermit, J., Barr, H., Garner, K., Rhoades, D., & Read, E. G. (1989). A Review of System Characteristics and Modelling Techniques for Multi-Reservoir Hydro-Thermal Electricity Systems.

OPUS. (2010). Lake Generation Potential History: Electricity Commission.

Pereira, M. V. F. (1989). Optimal stochastic operations scheduling of large hydroelectric systems. *International Journal of Electrical Power and Energy Systems, 11*(3), 161-169. doi: 10.1016/0142-0615(89)90025-2

Pereira, M. V. F., & Pinto, L. M. V. G. (1991). Multi-stage stochastic optimization applied to energy planning. *Mathematical Programming, 52*(1-3), 359-375. doi: 10.1007/BF01582895

Powell, W. B. (2007). *Approximate dynamic programming: solving the curses of dimensionality*. Hoboken, N.J: Wiley-Interscience.

Rani, D., & Moreira, M. M. (2010). Simulation-Optimisation Modeling: A Survey and Potential Application in Reservoir Systems Operation. *Water Resource Maagement, 24*, 1107-1138.

Read, E. G. (1979). *Optimal Operation of Power Systems.* (Doctor of Philosophy), University of Canterbury, Canterbury, New Zealand.

Read, E. G. (1984). Plant factors for oil-fired plant (Vol. no. ER 4007.). Wellington, N.Z. Ministry of Energy.

Read, E. G. (1985a). Notes on Electricity Planning Methods IV: Economically Balanced System Expansion. *Ministry of Energy Restricted Report, No. ER4019*

Read, E. G. (1985b). Notes on Electricity Planning Methods III: Long Term Reservoir Operating Rules. *Ministry of Energy Restricted Report, No. ER4017*

Read, E. G. (1985c). Notes on Electricity Planning Methods II: Computing Weekly Generation Schedules. *Ministry of Energy Restricted Report, No. ER4016*

Read, E. G., and Hindsberger, M. (2010). Constructive Dual DP for Reservoir Optimization. In S. Rebennack, Pardalos, P.M., Pereira, M.V.F., Iliadis, N.A. (Ed.), *Handbook of Power Systems 1* (Vol. 1, pp. 3-33): Springer.

Read, E. G., & George, J. A. (1990). Dual Dynamic Programming for Linear Production/Inventory Systems. *Journal of Computers and Mathematics, 19*(11), 29-42.

Read, E. G., & Yang, M. (1999). Constructive Dual DP for Reservoir Management with Correlation. *Water Resources Research, 35*(7), 2247-2257.

Read, R. A., Dye, S., & Read, E. G. (2012). *Generalized CDDP for Reservoir Management*. Paper presented at the Paper presented at the 46th Annual ORSNZ Conference, Victoria University, Wellington.

Scott, T. J., & Read, E. G. (1996). Modelling hydro reservoir operation in a deregulated electricity market. *International Transactions in Operational Research, 3*(3), 243-253. doi: 10.1016/S0969-6016(96)00019-6

Scott, T. J. a. R., E.G. (1996). Modelling hydro reservoir operation in a deregulated electricity market. *International Transactions in Operational Research, 3*(3), 243-253.

Stedinger, J., Sule, B., & Loucks, D. (1984). Stochastic dynamic programming models for reservoir operation optimization. *Water Resour. Res, 20*(11), 1499-1505.

Suvrajeet, S., & Zhihong, Z. (2014). Multistage Stochastic Decomposition: A Bridge between Stochastic Programming and Approximate Dynamic Programming. *SIAM Journal on Optimization, 24*(1), 127-153. doi: 10.1137/120864854

Tejada-Guibert, J., Johnson, S., & Stedinger, J. (1995). The value of hydrologic information in stochastic dynamic programming models of a multireservoir system. *Water Resour. Res., 31*(10), 2571–2579.

Trans Power New Zealand, L. (2008). HVDC Grid Upgrade Project.

Trans Power New Zealand, L. (2013). New HVDC Pole 3 Commissioned. https://www.transpower.co.nz/news/new-hvdc-pole-3-commissioned

Vasiliadis, H., & Karamouz, M. (1994). Demand-driven operation of reservoirs using uncertainty-based optimal operating policies. *J. Water Resour. Plan. Manage., 120*(1), 101-114.

Winter, N.L, E. G. Read, J. G. Culy, T. S. Halliburton. (1987). A Simulation Model for ELectricity Planning in New Zealand. *Operational Research, 98*, 493-507.

Wurbs, R. A. (1993). Reservoir-System Simulation and Optimization Models. *Journal of Water Resources Planning and Management, 119*(4), 455-472. doi: 10.1061/(ASCE)0733-9496(1993)119:4(455)

Wurbs, R. A., Tibbets, M. N., Cabezas, L. M., & Roy, L. C. (1985). State-of-the-art review and annotated bibliography of systems analysis techniques applied to reservoir operation. *TWRI Tech. Rep., 136*.

Yakowitz, S. (1982). Dynamic programming applications in water resources. *Water Resources Research, 18*(4).

Yang, M. (1995). *Dual Dynamic Programming For Reservoir Management With Correlated Inflows.* (Doctor of Philosophy), University of Canterbury, Canterbury, New Zealand.

Yeh, W. W. G. (1985). Reservoir Management and Operations Models: A State-of-the-Art Review. *Water Resources Research, 21*(12), 1797-1818. doi: 10.1029/WR021i012p01797

Appendix 1: Relationship of Efficiency to Losses

Representing losses in the system on an unconstrained line does not differ from the representation of electricity being made available at a different efficiency. With linear losses, then an unconstrained line with losses is effectively the same as all the generated energy being made available with a lower efficiency level. The key ratio in optimizing the system is the ratio of water released to electricity received, as opposed to the ratio of water released to electricity generated. This means that a 10% loss of efficiency during generation is indistinguishable in its impact from a 10% loss of delivered energy due to losses on the lines.

Furthermore, with non-linear losses approximated by a piecewise linear loss function, it would be possible to represent these non-linear losses as a number of efficiency tranches. Thus, through efficiency levels, the distinct sections of loss for a particular release are readily integrated into the system as a whole. An implementation of this is contained below in the case of two reservoirs existing in parallel but connected through a link with losses to the thermal generators filling the LDC.

Similarly, a constrained link would be identical to an unconstrained link except in that a release constraint would be applied to the DCRs before forming the DSRs. This would mean that a limited quantity of efficiency adjusted release could be released into the system in any one time period. Such a release limit would create a further constraint on the system as a whole, however there would be little additional computational requirement. This limited increase in the computation is due to the ability of the release limit to be incorporated into the DCR pre-computation which only occurs once for every reservoir configuration.